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A COMPARISON OF ROWLAND'S THERMOMETERS WITH THE PARIS STANDARD, AND A REDUCTION OF HIS VALUE OF THE MECHANICAL EQUIVALENT OF HEAT TO THE HYDROGEN SCALE.

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1. Introduction.

THE object of the investigation described here was to compare Professor Rowland's thermometers with the hydrogen scale of the International Bureau and to reduce his value of the mechanical equivalent accordingly.

The mechanical equivalent of heat, as it has been defined until recently, is, in the C. G. S. system, the value of J in the equation

$$J = \frac{dW}{dt}$$

where dW is the number of ergs required, when expended in heat, to raise the temperature of one gram of water $dt°$ C. on the absolute scale. The use of the expression "mechanical equivalent" in its old sense will be adhered to in this paper for the sake of uniformity with Rowland's paper. This quantity, $\frac{dW}{dt}$, differs at different temperatures on account of the variation of the specific heat of water.

Rowland's measurement of the mechanical equivalent, in 1877–78[1] still remains as probably the best one in which the heat was

[1] Proc. Am. Acad., 15, p. 75, 1879.

produced by the expenditure of mechanical energy. Within the last few years, however, a number of other determinations have been made by electrical methods, and these have given values differing somewhat from Rowland's. The chief of these electrical determinations are those of E. H. Griffiths[1] and of Schuster and Gannon.[2] The following table, taken from Schuster and Gannon's paper, give some comparative values. The unit is the foot-pound at Greenwich ($g = 981.17$ cm. / sec.^{2}) and the temperature scale is that of the Paris nitrogen thermometer.

	Joule.	Rowland.	Griffiths.	Schuster & Gannon.
At 19°.1 C	774	776.1	779.1	778.5
At 15°. C	775	778.3	780.2	779.7

The measurement of the quantity $\frac{dW}{dt}$ depends not only on the measurement of dW, but also on the scale of temperature used in measuring $dt°$. In Rowland's experiments this quantity was measured on the absolute scale, as determined by means of the air-thermometer. It was the object of the comparisons described here, to compare this absolute scale, as represented by Rowland's thermometers, with the hydrogen scale of the International Bureau of Weights and Measures, at Sèvres, near Paris. For this purpose there were secured three Tonnelot thermometers that had been studied at the International Bureau and compared with the standards of the Bureau at several points of the scale. These three thermometers were compared with three of Rowland's thermometers used in his mechanical equivalent experiment. The difference between Rowland's absolute scale and the Paris hydrogen scale was thus determined, and the corresponding corrections to the value of the mechanical equivalent were calculated. The general result is contained in Table IX. on page 218.

2. THERMOMETERS USED.

Rowland's value of the mechanical equivalent is the result of fourteen experiments, each of which was made with some one of four different thermometers in the calorimeter. In thirteen of these he used three thermometers made by Baudin in 1876–77, and in

[1] Phil. Trans., 184 A, p. 361, 1893.
[2] Phil. Trans., 186 A, p. 415, 1895.

one he used a thermometer made by Welsh in 1853, and called, in Rowland's paper, the Kew standard. This thermometer belongs to the University of Pennsylvania, and was not available for this comparison. The three Baudin thermometers were numbered 6163, 6165, and 6166. Of these, 6163 was used in eight experiments, 6165 in one experiment, and 6166 in four experiments. 6166 was also compared by Joule with one of his own thermometers.[1] The three Tonnelot thermometers were numbered 11,800, 11,801 and 11,811, and were made between November, 1895, and January, 1896. A complete study of these was made by the International Bureau, including a comparison at several points with the standards of the Bureau which had previously been compared with the hydrogen thermometer. Two of the thermometers, 11,800 and 11,811, were broken during their first transportation to America, but through the liberality and kindness of the International Bureau and of M. Guillaume, they were repaired by the maker, and a completely new study of them was made. These Tonnelot thermometers were made of French "verre dur," with transparent stems, and were divided into tenths of a degree. The Baudin thermometers were divided into millimeters. The centimeter marks were numbered, and corresponded in a rough way to degrees. The following table gives the dimensions and other data concerning these six thermometers.

TABLE I.

CONSTANTS OF THERMOMETERS.

Tonnelot Thermometers.			
Numbers.	11800	11801	11811
Divison.	.1°	.1°	.1°
Total length.	701 mm.	704 mm.	679 mm.
Middle of bulb to zero of scale.	55.8 mm.	59.0 mm.	74.3 mm.
Length of a "degree" of the scale.	5.86 mm.	5.858 mm.	8.91 mm.
Value, in degrees centigrade, of one "degree" of the scale.	0.870604°	1.000014°	0.977160°
Baudin Thermometers.			
Numbers.	6163	6165	6166
Division.	Millimeters.	Millimeters.	Millimeters.
Total length.	501 mm.	500 mm.	497 mm.
Middle of bulb to zero of scale.	34 mm.	37 mm.	34 mm.
Value, in degrees centigrade, of one centimeter on stem.	1.114°	0.8470°	0.779°

[1] Rowland, Proc. Am. Acad., 16, p. 38, 1880.

3. MANNER OF USING THE THERMOMETERS.

In comparing these various thermometers, it was necessary to use them in the way in which they were originally used when they were standardized. The Tonnelot thermometers had to be used in the manner described by Guillaume in his "Thermométrie de Précision," and in the printed notice that came with the thermometers from the International Bureau. Rowland's thermometers were used in a different way when they were originally compared with the air-thermometer. The chief points of difference are in the corrections for internal and external pressure, and in the manner of taking and using the zero point. The present method, as used at the International Bureau, is to determine the pressure coefficients, as they are called, by means of which any variation in the external pressure from the standard 760 mm., due to a change in barometric pressure, or to immersing the thermometer in a liquid, and any internal hydrostatic pressure of the mercury on the bulb, due to the thermometer not being horizontal, can be taken account of and the proper corrections applied. There are two pressure coefficients, the external and the internal. The external pressure coefficient is generally denoted by the letter β_e, and gives the fraction of a degree the reading of the thermometer is changed by a change in the external pressure of 1 mm. of mercury. The internal pressure coefficient, β_i, can be calculated for French hard glass by adding the quantity 0.0000154 to the external coefficient. This quantity depends on the elastic constants of mercury and of glass. It is generally a small fraction, 10 per cent. or less, of the whole pressure coefficient. It differs, of course, for different kinds of glass, but this difference is a still smaller fraction of the whole pressure coefficient.

The zero point is always calculated from the zero taken immediately after each reading of temperature, before the zero has had time to change.

Rowland's thermometers were originally used in quite a different way. In his comparisons, the vertical position was considered as the normal one, and the thermometers were always used in that position. Changes in external pressure were not taken account of, except when they were caused by the pressure of water in the comparison tank. The thermometers, when not in use, remained at a

temperature of 18° to 20° C., the temperature of the room. The zero points were taken at the beginning of a series of comparisons. In taking the zeros the thermometers were immersed in ice for an hour or two before the reading was taken, and then the comparisons were made at successively higher temperatures. In order to reduce the readings of his thermometers to the air thermometer, Rowland found by calibration the apparent volume occupied by the mercury corresponding to each division line on the stem. He assumed that the temperature on the air thermometer was a function of the second degree of this volume. He made a number of comparisons between his thermometers and the air thermometer, and,

TABLE II.

Reduction of Rowland's Baudin thermometers to the air thermometer, and to the absolute scale. From p. 115 of Rowland's paper on the mechanical equivalent of heat,

Column *A*. Reading in centimeters on the stem.

Column *B*. Temperature on the air thermometer.

Column *C*. Temperature on the absolute scale.

BAUDIN 6163.

A	*B*	*C*	*A*	*B*	*C*
5	−0°.911	−0°.911	24	20.350	20.345
5.81	0°	0°.	25	21.457	21.452
6	+0°.214	+0°.214	26	22.559	22.55[4]
7	1 .328	1 .328	27	23.657	23.652
8	2 .461	2 .460	28	24.755	24.750
9	3 .584	3 .583	29	25.854	25.848
10	4 .707	4 .706	30	26.956	26.950
11	5 .829	5 .827	31	28.060	28.056
12	6 .950	6 .948	32	29.169	29.163
13	8 .071	8 .069	33	30.282	30.276
14	9 .193	9 .190	34	31.398	31.392
15	10 .314	10 .311	35	32.514	32.508
16	11 .435	11 .432	36	33.630	33.624
17	12 .556	12 .553	37	34.748	34.742
18	13 .676	13 .672	38	35.864	35.85[7]
19	14 .794	14 .790	39	36.979	36.972
20	15 .909	15 .905	40	38.094	38.087
21	17 .022	17 .018	41	39.206	39.199
22	18 .132	18 .127	42	40.316	40.309
23	19 .242	19 .237			

TABLE II.—*Continued.*

BAUDIN 6165.

A	*B*	*C*	*A*	*B*	*C*
3 cm.	−0°.457	−0°.457	23 cm.	17.009	17.005
3.5	0°	0°	24	17.861	17.857
4	+0°.457	+0°.457	25	18.714	18.709
5	1 .368	1 .368	26	19.562	19.557
6	2 .276	2 .275	27	20.406	20.401
7	3 .174	3 .173	28	21.247	21.242
8	4 .069	4 .068	29	22.081	22.076
9	4 .957	4 .955	30	22.912	22.907
10	5 .841	5 .839	31	23.736	23.731
11	6 .714	6 .712	32	24.552	24.547
12	7 .590	7 .588	33	25.370	25.365
13	8 .459	8 .456	34	26.180	26.174
14	9 .324	9 .321	35	26.987	26.981
15	10 .186	10 .183	36	27.788	27.782
16	11 .042	11 .039	37	28.590	28.584
17	11 .896	11 .893	38	29.382	29.376
18	12 .749	12 .746	39	30.176	30.170
19	13 .601	13 .598	40	30.971	30.965
20	14 .453	14 .450	41	31.782	31.786
21	15 .305	15 .302	42	32.577	32.581
22	16 .157	16 .153			

by the method of least squares, found the equation connecting the temperature and volume. From this he constructed a table for each thermometer giving the air thermometer temperature corresponding to each centimeter on the stem. These tables are given on pp. 115 and 116 of his paper on the mechanical equivalent, and are reproduced here in Table II. The corrections to reduce the air thermometer temperature to the absolute scale are deduced from the experiments of Thomson and Joule, and are given here as Rowland gives them on p. 114 of his paper. (Table III.)

4. INSTRUMENTS EMPLOYED.

In making these comparisons, it would have been theoretically better to have used the Baudin thermometers vertically and the Tonnelot thermometers horizontally, or at least both thermometers

TABLE II.—*Concluded.*

BAUDIN 6166.

A	*B*	*C*	*A*	*B*	*C*
2 cm.	−0°.034	−0°.034	23 cm.	16.298	16.294
3	+0°.759	+0°.759	24	17.074	17.070
4	1 .553	1 .553	25	17.849	17.845
5	2 .336	2 .335	26	18.627	18.622
6	3 .115	3 .114	27	19.405	19.400
7	3 .889	3 .888	28	20.182	20.177
8	4 .665	4 .664	29	20.960	20.955
9	5 .438	5 .436	30	21.735	21.730
10	6 .212	6 .210	31	22.511	22.506
11	6 .988	6 .986	32	23.292	23.287
12	7 .765	7 .763	33	24.075	24.070
13	8 .544	8 .542	34	24.855	24.850
14	9 .323	9 .321	35	25.634	25.628
15	10 .105	10 .102	36	26.412	26.406
16	10 .887	10 .884	37	27.195	27.189
17	11 .667	11 .664	38	27.982	27.976
18	12 .444	12 .441	39	28.768	28.762
19	13 .217	13 .214	40	29.550	29.544
20	13 .988	13 .984	41	30.339	30.333
21	14 .756	14 .752	42	31.123	31.117
22	15 .526	15 .522	43	31.914	31.908

TABLE III.

REDUCTION OF AIR THERMOMETER TO ABSOLUTE SCALE.

Air thermometer.	Correction to Air thermometer.
0°	0°
10°	−0°.0028
20°	−0°.0048
30°	−0°.0061
40°	−0°.0067
50°	−0°.0068

vertically. But in order to make it possible to remove the Tonnelot thermometers easily and quickly from the comparison tank in order to take the zeros, and also in order that the readings of the thermometers might be made with the micrometer telescope, it was decided to make the comparisons with the thermometers in a horizon-

tal position, as otherwise there would have been serious mechanical difficulties in the construction of the tank. This made it necessary to apply a pressure correction to the Baudin thermometers to make the readings what they would have been if the thermometers had been vertical, but this correction could be applied with considerable accuracy by measuring the pressure coefficients.

In carrying out these comparisons, it was found necessary to construct several pieces of apparatus. As some of these differed a little from any that had been used before, it will be necessary to give a short description of them.

a. *Pressure Apparatus.*

The pressure coefficients were determined in a manner practically identical with that described by Guillaume in his "Thermométrie de Précision," p. 104. Matters were so arranged that the thermometer could be subjected alternately to a pressure of 5 or 6 cm. of mercury, and then to the atmospheric pressure of 76 cm. The change in pressure was read by means of a mercury manometer, and the change in the reading of the thermometer was also noted. The method of taking the readings will be described later.

b. *Micrometer Telescope.*

The thermometers, in the comparison tank and in the zero tank, were always read by means of a micrometer telescope. The micrometer was not an eye-piece micrometer, but there were cross-hairs in the field of view, and the whole telescope was moved parallel to itself by the micrometer motion. The telescope was so constructed that it could be used to read the horizontal thermometers in the comparison tank, or the vertical thermometers in the zero tank. In reading vertical thermometers, the telescope was fastened to an upright frame, consisting of two vertical steel bars. It then became in effect a cathetometer. In all cases care was taken by means of levels and plumb-lines to have the axis of the telescope perpendicular to the thermometers. The telescope magnified about ten or eleven times.

c. *Comparison Tank.*

The comparison tank was devised to fulfill the following requirements:

1. The thermometers must be easily removed for the purpose of taking zeros.

2. They must remain at a practically constant temperature for four or five minutes.

3. They must be easily and quickly read by means of a micrometer telescope sliding on ways above the tank.

4. The supports on which the thermometers rest must be so solid and firmly fastened to the ways on which the telescope slides that there will be no possibility of any accidental relative motion of the telescope and the thermometer.

To fulfill the first requirement, it was decided to have a horizontal tank, as explained farther back. The tank was made of sheet copper in the form of a rectangular box 92 cm. long, 21 cm. wide and 23 cm. deep. A section of it is represented in Fig. 1, and a cross-section in Fig. 2. The top consisted of a movable lid made of a brass casting. This lid had a flange that was planed on the under side so as to fit a planed brass rim forming the top of the sides of the tank. A piece of sheet India rubber formed a packing between the two parts, to prevent the escape of water. After the lid had been placed in position, it could be screwed down tightly by means of a number of C-clamps placed around the sides. The lid was made in the form of a frame holding a piece of plate glass, which was a window through which the thermometers could be read. This window was about 1.5 cm. from the stems of the thermometers.

In order to fulfill the second requirement, the tank was placed inside a larger wooden box whose walls were everywhere about 5 cm. from the sides of the tank, and the intervening space was filled with cotton wool. This protected the tank very well except at the top, from too rapid a gain or loss of heat from the air when this was at a different temperature from that of the tank. The layer of cotton did not extend quite up to the top of the tank. The non-conducting layer above this point had to be easily removed, to allow the removal of the lid. A covering of felt was therefore constructed, about 2 cm. thick, which extended over all the parts of the tank not protected by the cotton, except the glass window. This had to be exposed, at least in part, to allow of reading the ther-

mometers. In order still further to protect the thermometers from any change of temperature, the bulbs were placed inside a brass box, as shown in the sections. This box was 10.5 cm. long, 8 cm. wide and 5 cm. deep. It was so constructed that two lids on the top, and two on the bottom, could be opened or closed from the outside of the tank. While the water was being stirred, in the manner to be described presently, this box was kept open. As soon as the water had attained a uniform temperature, and everything was ready to make a comparison, the stirring was stopped, and the lids of the box were closed. The bulbs were thus shut off completely from any convection currents, and in this manner the thermometers could be kept at a constant temperature for a considerable time. The water was stirred by three paddle wheels operated by means of a crank on the outside of the box.

In order to heat the water in the tank to bring it to any desired temperature, a brass tube ran along it from end to end, near the bottom. This did not communicate with the water of the tank, but opened into the air at each end. Steam was passed into it until the water reached the temperature desired.

Fastened to the lid of the tank were upright brass posts which held the ways on which the micrometer telescope slid. The telescope had freedom to move in two horizontal directions at right angles to each other, so that it could be placed quickly over any part of the tank, and used to read any thermometer at any part of its scale. The thermometers rested on little brass beams which were fastened to the brass rim of the tank. As this rim was clamped securely to the brass lid which carried the ways, the fourth requirement was secured. When the tank was full of water and the lid on, the thermometer bulbs were about 5.7 cm. below the surface of the water.

d. *Zero Tank.*

In taking the zero points of thermometers in the ordinary tank, it it necessary to plunge the thermometer down into the ice until the zero division is a good deal below the surface, so as to make sure that the stem is all at the temperature of 0° C. When a reading is made, the thermometer is raised until the zero point is just visible.

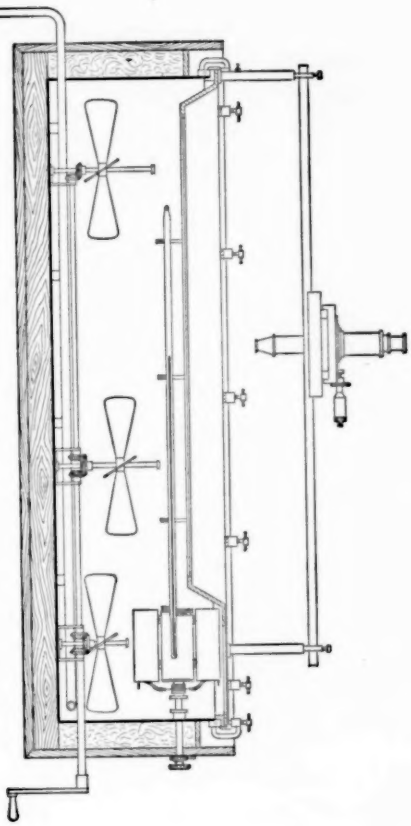

Fig. 1.—Section.

Comparison Tank.

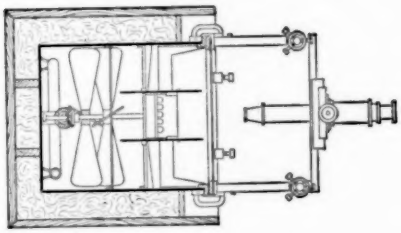

Fig. 2.—Cross Section.

When this is done, not only is the stem apt to become covered with water in films and drops, which make it difficult to read the position of the mercury thread, but it is also apt to become covered with dew when the laboratory room, as is usual in this country, is at a temperature of from 16° to 20° C. When the zero point is near the bulb, there is also, it seems probable, some danger that a little warm water may trickle down the stem of the thermometer and find its way to the bulb in sufficient quantity to make a difference of several thousandths of a degree. In order to do away with these difficulties, it was decided to have the stem immersed in the ice for some distance above the zero-point, and to read the thermometer through a tube which should reach from the side of the tank to the part of the stem where the zero-point actually was. A device was therefore employed which is represented in Fig. 3. The ice was contained in a cylindrical tin-lined copper tank, 33.5 cm. high, and 15.2 cm. in diameter. At the bottom there was an opening through which the excess of water could be drawn off. At a distance of 7 cm. from the top of the tank, two openings 2 cm. in diameter were made, one on each side of the tank. To these there was fitted, in a manner that it is not necessary to describe, but so that it could be readily detached, a tube of hard rubber, or ebonite, 2 cm. in diameter. This ebonite tube extended horizontally across the tank from one side to the other, except that at the center of it there was a little brass piece that held a sort of drum with drum-heads of pure sheet rubber. These were perforated with small circular holes in the center, and a thermometer could be inserted perpendicularly through them both. When in position the whole tank could be filled with ice and water, and very little if any water escaped through the joints. As represented in the drawing, the two rubber diaphragms, or drum-heads, were 3.7 cm. apart, and a portion of the thermometer stem of that length was, therefore, not exposed to the ice. This portion of the stem, however, must have been within 0°.1

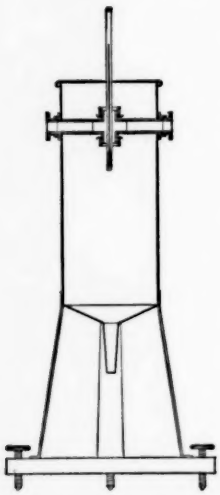

Fig. 3.—Zero Tank.

of 0° C., because when the bulb of a small thermometer which could detect a difference of 0°.1, was placed in the air in the center of this ebonite tube, and left there a short time, it indicated a temperature of 0° C. There was also no dew deposited on the stem unless there happened to be a draught of air blowing through the room, as happened in one or two cases.

In using the tank, the ice was broken up into very fine pieces by means of a wooden mallet in a zinc-lined wooden box, and transferred by a tin scoop to the zero tank. Ordinary distilled water was poured on it, and it was packed down closely. The thermometer was then placed in position, and the tank was filled to the brim with ice. The actual zero-point was generally observed in the middle of the tube. The water in the ice was also generally kept at about the same height. The actual height of the water was measured by means of a vertical glass tube placed alongside the tank, and communicating with the outlet tube at the bottom. A brass frame, not represented in the drawing, was fastened to the tank to support the thermometer.

The ice used was made artificially out of filtered Baltimore city water. Clear pieces were chosen, and it was found to be very pure by testing the water melted from it for ammonia, and for dissolved substances in general by measuring its electrical conductivity. The depression of the freezing point caused by impurities in the ice must have been less than 0°.001 C.

5. Pressure coefficients.

a. *Exterior Pressure Coefficient.*

In measuring the exterior pressure coefficient, precautions were taken, following Guillaume, to have the thermometer bulb practically surrounded by a large mass of water which was at a temperature several degrees cooler than that of the room. The water in this grew warmer continuously and with great uniformity. This prevented errors due to a falling meniscus, as readings were not taken immediately after the pressure and the reading of the thermometer were reduced. Readings were taken alternately at atmospheric pressure, and at a pressure of 5–10 cm. of mercury. The average of the pair at the high pressure of one atmosphere was

compared with the intervening reading at a low pressure. Ten or twenty observations of this kind were made for each thermometer, and the results averaged. In this way the pressure coefficient in centimeters on the stem was found. As the bores of the thermometer tubes were not very uniform in diameter, this was reduced to degrees centigrade by means of Rowland's tables.

The observations are given in Table IV. The results are as follows :

For Baudin 6163, $\beta_e = 0°.000274$.
For Baudin 6165, $\beta_e = 0°.000296$.
For Baudin 6166, $\beta_e = 0°.000318$.

b. *Interior Pressure Coefficient.*

In getting the interior pressure coefficient, the International Bureau, as mentioned before, calculates it from the external pressure coefficient. When the thermometer is made of French hard glass, the internal coefficient is connected with the external by the equation

$$\beta_i = \beta_e + 0.0000154$$

where the unit of measurement is degrees centigrade per millimeter of mercury pressure. Part of this difference depends on the cubical compression of mercury, and part on the elastic constants of the glass. The Baudin thermometers were not made of French hard glass, but it is probable that this difference would not be significantly changed by this fact. The same value has therefore been used. This gives for the three thermometers, the following values for the internal pressure coefficient.

For Baudin 6163, $\beta_i = 0°.000290$.
For Baudin 6165, $\beta_i = 0°.000311$.
For Baudin 6166, $\beta_i = 0°.000334$.

TABLE IV.

Observations for Pressure Coefficient.

Column *A*. Manometer reduced to 0° *C*. The pressure on the bulb of the thermometer = 1 atmosphere — the manometer reading.
Column *B*. Thermometer reading, in centimeters on the stem.
Column *C*. Means of pairs for manometer reading = 0 cm.
Column *D*. Observed differences, due to the changes in pressure.

BAUDIN 6163.

A	*B*	*C*	*D*
0	21.296 cm.		
72.38 cm.	21.121	21.300 cm.	0.179 cm.
0	21.305		
69.71	21.137	21.310	.173
0	21.315		
67.10	21.153	21.318	.165
0	21.322		
64.54	21.168	21.326	.158
0	21.331		
62.25	21.183	21.335	.152
0	21.340		
59.77	21.194	21.345	.151
0	21.350		
57.59	21.208	21.353	.145
0	21.357		
57.72	21.217	21.360	.143
0	21.364		
57.33	21.224	21.369	.145
0	21.374		
58.02	21.231	21.376	.145
0	21.378		
57.95	21.239	21.381	.142
0	21.385		
59.08	21.242	21.390	.148
0	21.395		
59.87	21.251	21.397	.146
0	21.400		
59.83	21.256	21.403	.147
0	21.407		
59.64	21.266	21.412	.146
0	21.417		
59.58	21.272	21.419	.147
0	21.422		
58.13	21.280	21.425	.145
0	21.428		
58.98	21.286	21.432	.146
0	21.436		
58.98	21.293	21.438	.145
0	21.441		
58.88	21.300	21.444	.144
0	21.447		
1217.33			3.012

12173.3 $\beta_e = 3.012$ cm.

$\beta_e = .000247$ cm. per mm. mercury pressure.

$\beta_e = 0°.000274$ per mm. mercury pressure.

TABLE IV.—*Continued.*

BAUDIN 6165.

A	*B*	*C*	*D*
0	23.088		
66.5	22.868	23.094	0.226
0	23.100		
64.3	22.885	23.109	.224
0	23.118		
62.2	22.909	23.127	.218
0	23.135		
61.3	22.928	23.142	.214
0	23.150		
60.4	22.948	23.157	.209
0	23.164		
59.8	22.965	23.171	.206
0	23.179		
59.4	22.982	23.187	.205
0	23.196		
59.2	23.000	23.202	.202
0	23.209		
58.5	23.015	23.218	.203
0	23.228		
58.6	23.033	23.235	.202
0	23.243		
58.4	23.049	23.251	.202
0	23.260		
58.0	23.065	23.268	.203
0	23.277		
58.9	23.079	23.285	.206
0	23.293		
59.4	23.093	23.302	.209
0	23.311		
59.3	23.114	28.320	.206
0	22.329		
59.1	23.131	23.338	.207
0	23.347		
58.9	23.150	23.357	.207
0	23.367		
59.8	23.166	23.375	.209
0	23.384		
59.3	23.186	23.393	.207
0	23.403		
1141.3			3.965

11413 $\beta_e = 3.965$ cm.

$\beta_e = 0.000347$ cm. per mm. mercury pressure.

$\beta_e = 0°.000296$ per mm. " "

TABLE IV.—*Concluded.*

BAUDIN 6166.

A	*B*	*C*	*D*
0	24.147		
61.1	23.900	24.153	.253
0	24.160		
59.0	23.927	24.171	.244
0	24.183		
56.8	23.955	24.190	.235
0	24.198		
56.2	23.980	24.208	.228
0	24.218		
56.0	24.000	24.228	.228
0	24.238		
55.5	24.014	24.245	.231
0	24.253		
53.5	24.038	24.258	.220
0	24.263		
51.6	24.062	24.270	.208
0	24.278		
49.8	24.084	24.286	.202
0	24.294		
48.0	24.109	24.308	.199
0	24.322		
547.5			2.248

5475 $\beta_e = 2.248$ cm.
$\beta_e = .000411$ cm. per mm. mercury pressure.
$\beta_e = 0°.000318$ per mm. " "

6. MANNER OF MAKING THE COMPARISONS.

In comparing a Tonnelot with one of the Baudin thermometers, the Baudin was placed first in ice for two or three hours. Its zero point was then read by means of the micrometer telescope three or four times in succession, and the average taken as the reading. In the meantime the water in the comparison tank having been cooled to about 10° or lower, the Baudin and one of the Tonnelot thermometers were placed in the tank. The top of the tank was put on, and while the box containing the bulbs re-

mained open, the water was thoroughly stirred. As long as the temperature of the water in the tank was lower than that of the room, it of course rose very slowly while the stirring went on. This sufficed to make the thermometers always reach any given temperature below that of the room with a rising meniscus. As soon as the water was thoroughly mixed, the stirring was stopped, and the inner box immediately closed. A micrometer reading of the mercury thread on one of the thermometers was then taken quickly, the telescope shifted quickly to the other thermometer, and a reading taken of it. A second reading of both thermometers was also taken. Of course, the first readings were the most reliable, but when there was no delay, there was hardly ever any appreciable change of temperature observed on reading the second time, except sometimes at the extreme temperature of 40°. The second reading was taken more as a check on the first, to make sure there was no error in reading. Having made this observation at a certain temperature, the inner box was opened, the water stirred up again, and another comparison made in the same way. This would be, of course, at a slightly higher temperature. These readings at two neighboring temperatures were averaged, and the average regarded as one reading. When the temperature was higher than that of the room, the only difference made was that steam was passed into the heating tube just before a reading was taken. The water was stirred, and the thermometers were brought to the position in which they were read with a rising meniscus, and not with a falling one. This was made very sure by observing one of the thermometers through the telescope while the heating and stirring was going on. Having made two observations at neighboring temperatures, the top of the tank was taken off, the Tonnelot thermometer removed, and its zero determined. The Tonnelot thermometer was then replaced, the top of the tank put on, the water heated, and a comparison made in the same way at the next temperature. It was not possible conveniently to make comparisons at many points in one day. The intention was, therefore, to take observations roughly at 10°, 20°, 30° and 40°. This was only approximately carried out, as it sometimes became convenient, for a good many reasons, to use temperatures intermediate to these.

7. Comparisons Made.

A number of comparisons were made between the Tonnelot thermometers taken in pairs, between the Tonnelot and Baudin thermometers, and between the Baudin thermometers. The results of these comparisons are given in Tables V. and VII. The comparisons may seem to be rather few in number, but it was not thought worth while to make any more, considering all the circumstances. It must be remembered that the readings of the thermometers were made micrometrically, so that the errors in reading are exceedingly small. Such large experimental errors as there are, must be probably in connection with the position of the zero. Rowland always read his thermometers with the naked eye, and although this was the only practicable way of using them in his experiment, the fact must be considered in making any comparisons between his thermometers and any others.

Two comparisons were made between Tonnelot 11,801 and Tonnelot 11,811, and three between Tonnelot 11,800 and Tonnelot 11,811. In one of the latter comparisons, that of 21 April, the thermometer 11,811 became accidentally heated up to 30° before a reading at 20° was taken, and for this reason, on account of the recovery of the zero being in progress while the comparison was being made, it was thought probable that some irregularity would be shown. This proved to be the case, although it is not certain that the irregularity was due to the overheating. There were also other disturbing influences that made it seem probable at the time that the comparison would not be reliable. The comparison, however, was rejected. In comparing the Tonnelot thermometers, they were always placed in ice for some hours before a comparison was begun, in case the first temperatures used were lower than the temperature to which the thermometer had been exposed for some time previously, so that the recovery of the zero might have a chance to proceed as far as possible before the comparisons were begun. The results of the comparisons of the Tonnelot thermometers with each other are given in Table V. The two comparisons of each pair seemed to agree so well that, without taking any further time to make more comparisons, it was decided that they represented substantially the difference between the thermometers. From these

comparisons smooth correction curves were drawn, so that the readings of the thermometers 11,800 and 11,811 could be reduced to those of 11,801. These corrections, however, are small, amounting in most cases to only a few thousandths of a degree. A table of these corrections, deduced from the smooth curves, is given in Table VI.

TABLE V.

Comparisons of Tonnelot thermometers with each other.

11801 *and* 11811 *compared.*

Observed readings.		Readings reduced to the hydrogen scale.		
Tonnelot. 11801	Tonnelot. 11811	Tonnelot. 11801	Tonnelot. 11811	11801 −11811
		12 April, 1897.		
10°.529	10°.615	10°.421	10°.423	−°.002
25 .470	25 .981	25 .338	25 .344	− .006
36 .643	37 .354	36 .478	36 .475	+ .003
		14 April, 1897.		
7°.681	7°.682	7°.570	7°.570	°.000
24 .562	25 .051	24 .432	24 .436	− .004
36 .725	37 .438	36 .560	36 .559	+ .001

11800 *and* 11811 *compared.*

Tonnelot. 11800	Tonnelot. 11811	Tonnelot. 11800	Tonnelot. 11811	11800 −11811
		21 April, 1897.		
7°.196	8°.019	7°.898	7°.899	−°.001
20 .180	19 .610	19 .143	19 .140	+ .003
29 .925	28 .324	27 .598	27 .607	− .009
38 .783	36 .165	35 .297	35 .308	− .011
		23 April, 1897.		
9°.523	10°.079	9°.896	9°.896	°.000
21 .232	20 .554	20 .053	20 .059	− .006
30 .961	29 .246	28 .500	28 .504	− .004
40 .879	38 .022	37 .123	37 .132	− .009
		27 April, 1897.		
7°.966	8°.702	8°.570	8°.568	+°.002
20 .299	19 .725	19 .252	19 .257	− .005
32 .595	30 .689	29 .927	29 .933	− .006
40 .773	37 .929	37 .035	37 .041	− .006

TABLE VI.

Corrections for reducing the readings of Tonnelot 11800 and 11811 after reduction to the hydrogen scale, to the hydrogen scale as represented by Tonnelot 11801. These corrections are the ordinates of the smooth curves which were deduced from the comparisons given in Table V.

Temperature.	$T_{01}-T_{00}$	$T_{01}-T_{11}$
2°	0	0
4	0	0
6	−0.001	−0.001
8	−0.001	−0.001
10	−0.001	−0.001
12	−0.001	−0.002
14	0	−0.002
16	0	−0.003
18	+0.002	−0.003
20	+0.001	−0.004
22	+0.001	−0.005
24	0	−0.005
26	+0.001	−0.004
28	+0.002	−0.003
30	+0.003	−0.002
32	+0.005	−0.001
34	+0.006	0
36	+0.008	+0.001

TABLE VII.

Comparisons between Tonnelot and Baudin thermometers.

Observed Readings.		Reduced Readings.		Differences.
Tonnelot.	Baudin.	Tonnelot. *H*— scale.	Baudin. Abs. scale.	*T*—*B*
		23 March, 1897.		
		Tonnelot 11801 *and Baudin* 6163.		
At beginning, observed zero of Baudin 6163		6.021 cm.		
" " reduced zero of Baudin 6163		+0°.238.		
7°.055	12.236 cm.	6°.941	6°.932	+0°.009
14 .161	18.598	14 .050	14 .041	+ .009
25 .335	28.708	25 .202	25 .198	+ .004
33 .044	35.678	32 .910	32 .916	− .006
40 .509	42.364	40 .328	40 .344	− .016
		29 March, 1897.		
		Tonnelot 11811 *and Baudin* 6163.		
At beginning, observed zero of Baudin, 6.029 cm.				
" " reduced zero of Baudin, +0°.243.				
10°.329	15.106 cm.	10°.141	10°.131	+0°.010
18 .166	21.928	17 .743	17 .729	+ .014
29 .969	32.364	29 .226	29 .219	+ .007
37 .227	38.777	36 .357	36 .356	+ .001

TABLE VII.—*Continued.*

Comparisons between Tonnelot and Baudin Thermometers.

Observed readings. Tonnelot.	Baudin.	Reduced readings. Tonnelot. *H*— scale.	Baudin. Abs. scale.	Differences. *T—B*
		31 March, 1897.		
		Tonnelot 11800 *and Baudin* 6163.		
At beginning, observed zero of Baudin, 6.021 cm.				
" " reduced zero of Baudin, +0°.234.				
9°.361	14.756 cm.	9°.755	9°.748	+0°.007
20 .231	23.228	19 .188	19 .176	+ .012
30 .385	31.255	28 .002	28 .001	+ .001
39 .410	38.320	35 .846	35 .857	− .011
		24 March, 1897.		
		Tonnelot 11801 *and Baudin* 6165.		
At beginning, observed reading of Baudin, 3.823 cm.				
" " reduced zero of Baudin, +0°.299.				
10°.442	15.586 cm.	10°.338	10°.331	+0°.007
19 .568	26.327	19 .454	19 .446	+ .008
28 .854	37.731	28 .737	28 .740	− .003
		26 March, 1897.		
		Tonnelot 11811 *and Baudin* 6165.		
At beginning, observed zero of Baudin 6165. 3.818 cm.				
" " reduced zero of Baudin 6165. +0°.290.				
9°.624	14.565 cm.	9°.467	9°.461	+ .006
22 .358	29.161	21 .817	21 .818	− .001
30 .301	38.756	29 .551	29 .554	− .003
		1 April, 1897.		
		Tonnelot 11800 *and Baudin* 6165.		
At beginning, observed zero of Baudin 6165. 3.824 cm.				
" " reduced zero of Baudin 6165. +0°.292.				
10°.971	16.530 cm.	11°.153	11°.133	+ .020
18 .209	23.930	17 .434	17 .415	+ .019
31 .533	38.062	29 .004	28 .999	+ .005
		24 March, 1897.		
		Tonnelot 11801 *and Baudin* 6166.		
At beginning, observed zero of Baudin 6166 2.420 cm.				
" " reduced zero of Baudin 6166 +0°.305.				
10°.521	15.844 cm.	10°.417	10°.399	+0°.018
19 .573	27.544	19 .459	19 .420	+ .039
28 .645	39.242	28 .528	28 .509	+ .019
		26 March, 1897.		
		Tonnelot 11811 *and Baudin* 6166.		
At beginning, observed zero of Baudin 6166 2.417 cm.				
" " reduced zero of Baudin 6166 +0°.297.				
9°.613	14.610 cm.	9°.456	9°.440	+ .016
21 .795	29.896	21 .270	21 .241	+ .029
30 .190	40.415	29 .442	29 .428	+ .014

TABLE VII.—*Concluded.*

Comparisons between Tonnelot and Baudin thermometers.

Observed Readings.		Reduced Readings.		Differences.
Tonnelot.	Baudin.	Tonnelot. *H* — scale.	Baudin. Abs. scale.	*T*—*B*
		1 April, 1897.		
		Tonnelot 11800 *and Baudin* 6166.		
At beginning, observed zero of Baudin 6166		2.418 cm.		
" " reduced zero of Baudin 6166		+0°.293.		
10°.861	16.664 cm.	11°.057	11°.038	+ .019
18 .171	24.896	17 .402	17 .372	+ .030
31 .143	39.417	28 .664	28 .648	+ .016
		26 April, 1897.		
		Tonnelot 11801 *and Baudin* 6163.		
At beginning, observed zero of Baudin 6163		6.006.		
" " reduced zero of Baudin 6163		+0°.221.		
9°.539	14.454 cm.	9°.432	9°.427	+ .005
19 .674	23.550	19 .555	19 .548	+ .007
30 .825	33.691	30 .707	30 .721	− .014
39 .512	41.454	39 .331	39 .354	− .023

Next a series of comparisons was made between each Tonnelot thermometer and each Baudin. The results of these comparisons are given in Table VII., p. 213. The readings of the Tonnelot thermometers were reduced to the hydrogen scale, applying all the corrections supplied by the International Bureau, and the corrections mentioned in the last paragraph, required to reduce the readings to those of Tonnelot 11801. Tonnelot 11801 was chosen for this purpose because it was the one which had not been broken. Each of these comparisons gave a correction curve, the ordinates of which represented the corrections that must be applied to the readings of the Baudin thermometers, when reduced to the absolute scale, to reduce them to the hydrogen scale, as represented by the Tonnelot thermometers. From these three curves for each Baudin thermometer there was deduced an average correction curve for each. The ordinates of this final smooth curve for each Baudin thermometer are given in Table VIII., p. 216. As there were three thermometers of each kind, there were nine possible compari-

TABLE VIII.

Corrections to be added to the readings of the Baudin thermometers, when they have been reduced to the absolute scale, to reduce them to the Paris hydrogen scale, as represented by Tonnelot 11801. These corrections are the ordinates of the smooth curves which were deduced from the comparisons given in Table VII. The corrections are given in thousandths of a degree.

Temp.	6163	6165	6166	Temp.	6163	6165	6166
1°	+ 1	+ 1	+ 2	21°	+10	+ 8	+31
2	2	3	3	22	9	7	30
3	3	4	5	23	8	6	29
4	4	5	6	24	7	5	27
5	5	6	8	25	6	4	25
6	6	7	10	26	5	3	23
7	6	8	12	27	4	2	21
8	7	9	13	28	3	1	19
9	8	10	15	29	2	0	16
10	9	11	17	30	1	−1	13
11	9	11	19	31	− 1	−2	10
12	10	12	21	32	− 2	−3	7
13	10	11	23	33	− 3	−4	
14	11	11	25	34	− 5		
15	11	11	27	35	− 6		
16	11	10	29	36	− 7		
17	11	10	30	37	− 8		
18	11	9	31	38	− 9		
19	11	9	32	39	−10		
20	10	8	32	40	−11		

sons. These nine were all made, and finally an additional one on April 26, 1897, between Tonnelot 11801 and Baudin 6163. When this was made the Baudin thermometer had not yet recovered its zero from a depression caused by frequent use up to within six days of the time of the comparison. Its correction curve therefore differed from the others, but in the way one would expect on account of this depression. It was, however, rejected. The corrections given in Table VIII. were finally used to calculate the changes to be made in Rowland's value of the mechanical equivalent to reduce it to the Paris hydrogen scale.

In order to get a further check on the comparisons, the three Baudin thermometers were placed in the tank at the same time, and

compared together. Without going into the details here, it may be said that these comparisons verified the results, so far as it was possible, of the other comparisons.

8. Calculation of the Effect on Rowland's Value of the Mechanical Equivalent of Heat.

In order to estimate the effect of these corrections to the Baudin thermometers on the value of the mechanical equivalent of heat, the experiments made by Rowland with each thermometer were considered separately. Rowland has a table on p. 192 of his paper giving the mechanical equivalent, in kilogram-meters at Baltimore, each value calculated from a rise of 10° in temperature, as deduced from each of his experiments. From this table the mean value of the mechanical equivalent at each temperature was calculated, as obtained by each thermometer separately. Rowland's value was deduced from an equation of the form

$$J = \frac{W}{10^\circ}$$

where W is a certain number of ergs, found by experiment, required to raise the temperature of 1 gram of water 10° C. J at 12°, for example, was calculated from the value of W in going from 7° to 17° C. That is

$$J_{12} = \frac{W}{17^\circ - 7^\circ}$$

If C_{17} and C_7 are the corrections found for the readings of the Baudin thermometer, to reduce them to the hydrogen scale, these corrections, applied to the value of J_{12} will give the corrected value of J_{12}, which shall be called J_{12}'. This gives

$$J_{12}' = \frac{W}{17^\circ + C_{17} - (7^\circ + C_7)} = \frac{W}{10^\circ + C_{17} - C_7}$$

$$\therefore J_{12}' = \frac{W}{10} \cdot \frac{10}{10 + C_{17} - C_7} = J_{12} \frac{10}{10 + C_{17} - C_7}$$

When $C_{17} - C_7$ is very small, as is always the case in these corrections, this may be written

$$J_{12}' = J_{12}\left(1 - \frac{C_{17} - C_7}{10} + \ldots\right) = J_{12} + J_{12}P_{12}$$

where $$P_{12} = \frac{C_7 - C_{17}}{10}$$

This conversion factor, P_{12}, was calculated similarly for each degree of the scale where it was necessary, and for each thermometer, and the corresponding value of J' calculated. Then taking account of the number of observations on each thermometer, used in getting the average value of J, these corrected values were averaged for each degree at which observations were made. The resulting values are given in Table IX. These values were plotted, and a smooth curve drawn in the neighborhood of the points. The ordinates deduced from this smooth curve give the corrected value of the me-

TABLE IX.

General result of comparisons.

Column *A*. Temperatures on absolute scale.
Column *B*. Corrected values of the mechanical equivalent of heat, hydrogen scale, kilogrammeters at Baltimore, from smooth curve.
Column *C*. Same, converted to the C. G. S. system.
Column *D*. Old values, from Rowland's paper, Table 54, p. 196.

A	*B*	*C*	*D*	*A*	*B*	*C*	*D*
5°			4212	21°	426.5	4180	4177
6	428.9	4203	4209	22	426.4	4179	4176
7	428.7	4201	4207	23	426.3	4178	4175
8	428.5	4199	4204	24	426.2	4177	4174
9	428.3	4198	4202	25	426.1	4176	4173
10	428.1	4196	4200	26	426.1	4176	4172
11	427.9	4194	4198	27	426.0	4175	4171
12	427.7	4192	4196	28	426.0	4175	4171
13	427.6	4191	4194	29	425.9	4174	4170
14	427.4	4189	4192	30	425.9	4174	4171
15	427.3	4188	4189	31	425.9	4174	4171
16	427.1	4186	4187	32	425.9	4174	4171
17	427.0	4185	4185	33	425.9	4174	4172
18	426.9	4184	4183	34	425.9	4174	4172
19	426.7	4182	4181	35	426.0	4175	4173
20	426.6	4181	4179	36	426.0	4175	4173
		$\times 10^4$	$\times 10^4$			$\times 10^4$	$\times 10^4$

chanical equivalent in kilogram-meters at Baltimore, as deduced from the comparisons described here. These values are given in Table IX. in kilogram-meters, and also in the C. G. S. system. They are also shown in the curves in Fig. 4. On this diagram

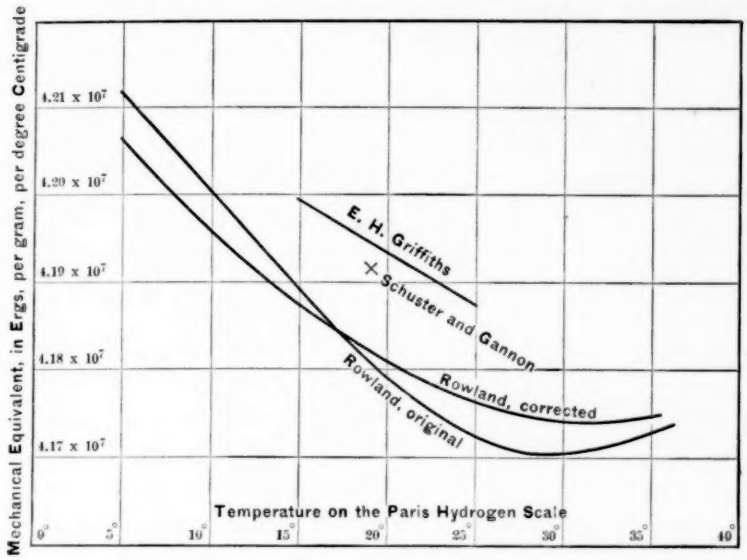
Mechanical Equivalent, in Ergs, per gram, per degree Centigrade
4.21 x 10⁷
4.20 x 10⁷
4.19 x 10⁷
4.18 x 10⁷
4.17 x 10⁷
E. H. Griffiths
Schuster and Gannon
Rowland, corrected
Rowland, original
Temperature on the Paris Hydrogen Scale
0° 5° 10° 15° 20° 25° 30° 35° 40°

Fig. 4.

are drawn curves representing Rowland's uncorrected values, the corrected values, as given in Table IX., and the values obtained by E. H. Griffiths, after correcting them to the hydrogen scale.[1] Griffiths' values were corrected to the hydrogen scale from the nitrogen scale by means of the differences between the two scales, as given by the experiments of Chappuis.[2] Schuster and Gannon's value, 4.1917×10^7 at 19°.1, is also marked on the same diagram for comparison. [3]

9. Schuster's Indirect Comparison of Baudin 6166 with the Paris Nitrogen Thermometer.

Before the comparisons described here were made, one of Rowland's thermometers had already been compared indirectly with

[1] Phil. Trans., 186 A, p. 361, 1893; Phil. Mag,, 40, 447 and 437, 1895.

[2] Guillaume, Thermométrie de Précision, p. 258.

[3] Phil. Trans., 186 A, p. 458, 1895.

the Paris nitrogen thermometer. In the summer of 1879, Joule himself made a comparison of Rowland's thermometer, Baudin 6166, with his own thermometer, and the results were published by Rowland.[1] In 1892–94, Schuster made a comparison of Joule's thermometer with a Tonnelot that had been standardized at the International Bureau, and he reduced the indications of the Joule thermometer to the Paris nitrogen scale.[2] These two comparisons should give a means of reducing the indications of Rowland's thermometer to the Paris nitrogen or hydrogen scale. The present writer has made an examination of the comparisons of Schuster and of Joule, and, by means of a graphic method, the results obtained by Schuster's indirect comparison were compared with those obtained from the present series. The curves drawn are shown in Fig. 5. The notation used is intended to be suggestive of the meaning of the curves. The ordinates of the

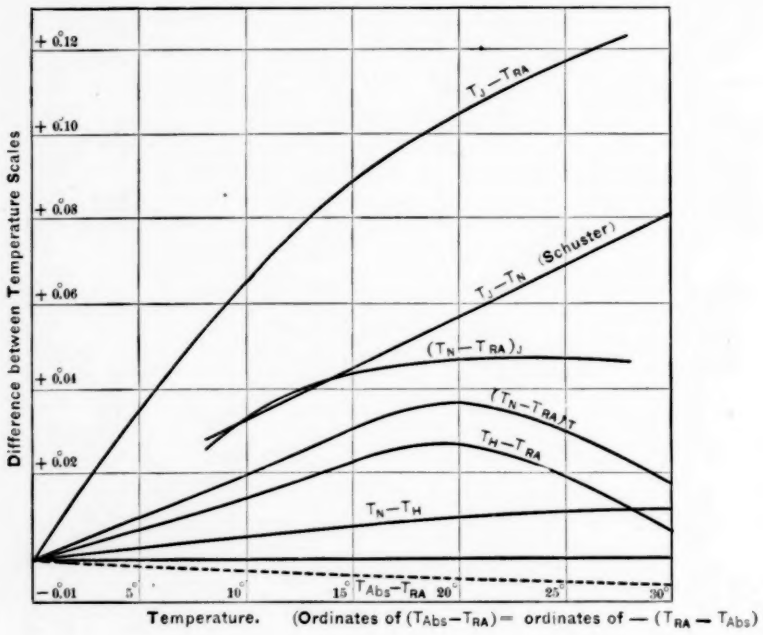

Fig. 5.

[1] Proc. Am. Acad., Vol. 16, p. 38, 1880–81.

[2] Schuster, Phil. Mag., Vol. 39, p. 477, 1895.

curve marked $T_{RA} - T_{Abs}$ give the differences between the air thermometer and the absolute scale, as used by Rowland. The ordinates of the curve $T_N - T_H$ represent the differences between the nitrogen and the hydrogen scale, according to Chappuis. The ordinates of the curve $T_H - T_{Abs}$, which is not shown in Fig. 5, represent the differences between the hydrogen scale and Rowland's absolute scale according to Baudin 6166, as given by the present series of comparisons. (See Table VIII.) In order to get the ordinates of the curve $(T_N - T_{RA})_T$, which represents the differences between the Paris nitrogen scale and Rowland's air thermometer scale according to Baudin 6166 and the Tonnelot thermometers, the ordinates of the curves were added, as represented by the following equation:

$$(T_H - T_{Abs}) - (T_{RA} - T_{Abs}) + (T_N - T_H) = (T_N - T_{RA})_T$$

In a similar manner the curve $T_J - T_{RA}$ is the smooth curve which represents approximately the differences found by Joule between his thermometer and Rowland's air thermometer, according to Baudin 6166. The curve $T_J - T_N$, which is a straight line, is taken to represent the differences between Joule's thermometer and the Paris nitrogen thermometer, according to the comparisons of Schuster. Schuster does not give this curve, but merely its slope. If the points found by his experiments are plotted on section paper, the straight line

$$(T_J - T_N) = 0°.0024\ T_N + 0°.009$$

which has the slope he gives, seems most nearly to represent the average of the points. The final curve $(T_N - T_{RA})_J$ is found by subtracting ordinates as represented by the equation

$$(T_J - T_{RA}) - (T_J - T_N) = (T_N - T_{RA})_J$$

As shown in Fig. 5, this curve is not very unlike the curve $(T_N - T_{RA})_T$. The maxima are near 20° in both, and the difference between the two is about 0°.01, which is not very great in view of the ignorance as to how Joule made his comparisons, and the chances of a weak point in so long a chain.

10. Conclusion.

These comparisons do not explain the discrepancy between Rowland's value of the mechanical equivalent of heat, and the higher val-

ues found by recent experimenters using electrical methods. They seem to show, however, that the discrepancy is not one of thermometry. It is probable, therefore, that it is due to some error in the measurement of energy in Rowland's experiments, or to a similar error in the electrical experiments. In Griffiths' measurement of electrical energy expended in heat, the square of the electromotive force enters as a factor in the value of J. This makes any error in the measurement of this quantity have double the effect on the value obtained for J. An error in the standard of electromotive force of about one-tenth per cent. would, therefore, suffice to account for the entire difference between Rowland's results and Griffiths'.

The most interesting result of the present comparison is that Rowland's corrected curve shows the same rate of change in the specific heat of water with the temperature between 15° and 25° that Griffiths' does, as can be seen at a glance by an inspection of the curves as drawn in Fig. 4. This fact also suggests the possibility of the difference being due to an error in the standards used, rather than in the experiments themselves.

A short abstract of the results of these comparisons was published in the Johns Hopkins University Circular for June, 1897, giving the corrected values of the mechanical equivalent of heat. There was a slight error in the values as published, owing to the use of an incorrect method of reducing the readings of Rowland's Baudin thermometers. The error was not discovered until after the abstract had been published. This incorrect abstract was reproduced in the *Philosophical Magazine* for August, 1897.

The author wishes to express his appreciation of Professor Rowland's kindness in allowing the use of his thermometers for these comparisons, and his obligation to Professor Rowland and Professor Ames for their frequent aid and advice in the course of the work.

Although the general plan of the apparatus described here is the author's, he wishes to express his deep sense of obligation to Dr. W. T. Mather, at that time Fellow of Johns Hopkins University, for his aid at almost every step in the mechanical details of its design and construction.

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ON THE SUSCEPTIBILITY OF DIAMAGNETIC AND WEAKLY MAGNETIC SUBSTANCES.

By Albert P. Wills.

FARADAY (1845) showed that all substances, whether solid, liquid or gaseous, are either diamagnetic or magnetic. Previously, however, Brugmans (1778) and Becquerel (1827) observed certain diamagnetic phenomena.

By way of relative measurement of the coefficient of susceptibility, a great deal has been done by various physicists, among them Plücker and Faraday. For bismuth there have been made a number of determinations in absolute measure. It will suffice to mention here four of the best methods used: 1. The method used by Christie[1] (1858) depends upon the comparison of the inductive action of the magnetised bismuth with that of a solenoid through which a known current is flowing. 2. The method used by Töpler and von Ettingshausen[2] (1877) depends upon the measurement of the force of repulsion upon a piece of bismuth hung near the end of a solenoid. 3. The method used by Rowland and Jacques[3] (1879) depends upon the measurement of the influence of the diamagnetic force upon the time of swing of a piece of bismuth suspended in an inhomogeneous magnetic field. 4. The method used by von Ettingshausen[4] (1882) depends upon the measurement directly of diamagnetic action at a distance.

Von Ettingshausen has used all of the above mentioned methods, and for an interesting comparison of results obtained with these methods, a table, taken from Winkelmann's Handbuch der Physik, is subjoined:

[1] Christie, Pogg. Ann., 103, p. 577, 1858.
[2] Töpler u. v. Ettingshausen, Pogg. Ann., 160, p. 1, 1877.
[3] Rowland and Jacques, Am. Jour. Sci., 18, p. 360, 1879.
[4] v. Ettingshausen, Wien. Ber., 85 (2) p. 37; Wied. Ann., 17, p. 272.

VALUES OF κ FOR BISMUTH.

Observer.		Extreme Values.	Mean Values.
v. Ettingshausen	1 Method :	13.35 to 13.82 x 10–6	13.57 x 10–6
"	2 "	13.82 to 14.08 "	13.99 "
"	2 "	14.41 to 14.80 "	14.54 "
"	2 "	13.41 to 13.54 "	13.48 "
"	2 "	14.09 to 14.13 "	14.11 "
"	3 "		15.30 "
"	4 "	13.13 to 13.66 "	13.35 "
"	General Mean		14.05 "

Von Ettingshausen also made absolute determinations of the susceptibility for antimony and tellurium. There has been a considerable amount of work done by various physicists by way of determination of the susceptibility for liquids and gases. A great number of liquids and gases have been used and the susceptibility has been determined for them by different methods. The results thus obtained sometimes differ widely.

Concerning the determination of the susceptibility for bodies in which it is exceedingly small, it may be said that the methods previously used have sometimes suffered greatly in consequence of the weakness of the action to be observed. No doubt many of the divergent results obtained by different physicists are due in large measure to dissimilarity in the samples of the substance experimented upon. In some cases, however, they seem due to some chronic weakness in the method of experimentation. A fundamental difficulty met with in the elaboration of a method for the study of diamagnetic phenomena lies in making the inevitably small action as large as possible.

In the autumn of 1895,[1] acting upon the suggestion of Professor A. G. Webster, I attempted to determine the susceptibility of substances by suspending an ellipsoid of the substance in a homogeneous magnetic field and observing the times of oscillation with a known

[1] In connection with this work I noticed a very remarkable behavior of non-conductors in a magnetic field, which led me to suspect an action akin to hysteresis in iron. As my direct object at that time was to establish a method for the determination of coefficients of susceptibility I did not push my experiments far enough to warrant my publishing results. Mr. Duane has published (Wied. Ann., Bd. 58, p. 517, 1896) an account of similar experiments performed by him in the physical laboratory at Berlin, which seemed to indicate hysteresis in non-conductors, but in which later research showed the effect to be due to traces of iron. (Wied. Ann. 61, p. 436, 1897.)

field and with no field. I failed in this object owing to the fact that I was unable to obtain a field which was uniform to a sufficient degree of approximation, the effect which I wished to observe depending upon the square of the extremely small quantity $\varkappa$ while the effect depending upon the non-uniformity of the field varies with the first power of $\varkappa$.

The purpose of the present paper is to describe a method for the determination of $\varkappa$ for bodies in which it is extremely small and either positive or negative, and to give the actual results obtained from experiments upon a large number of substances.

Description of the Apparatus.

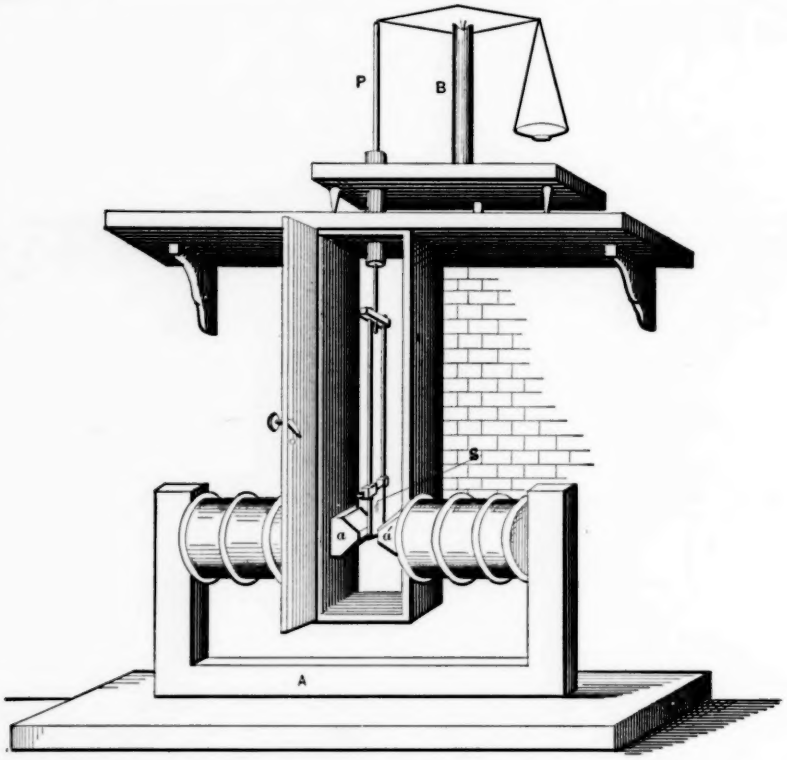
It has long been known that if any substance be introduced into an inhomogeneous magnetic field it experiences a mechanical force which tends to make it enter a stronger or weaker part of the field, according as the substance is magnetic or diamagnetic. To determine this force the amount and distribution of the field in which the substance is placed must be known.

I shall describe the apparatus used in the present experiments and in the theory of the method shall show that the apparatus, as here described, is adequate to allow for certain simplifications in the general theory, giving the force acting on a body placed in an inhomogeneous magnetic field. As a result of these simplifications we get an expression for the mechanical force upon the body in a convenient direction, which involves the field strength at the two limiting surfaces only, in that direction. A suitable form for the experimental body is that of a thin rectangular slab. Fig. 1 will give a general idea of the arrangement of the apparatus.

The pole pieces are prismatic, as shown in Figs. 2 and 3. The bevelled faces are on angles of 60 degrees. When in position the pole forces are parallel and the gap between them is about 1.5 cm. The figures given on the diagram represent centimeters.

In Fig. 4 is shown the suspension arrangement used in holding the slab. P is the suspension wire from the balance beam passing through the vulcanite cross-arm c. W and W' are rigid brass wires passing easily through slots in c and supported by means of nuts d and d', the upper ends of W and W' being threaded; b and b' are

small vulcanite blocks to which the rigid wires W and W' are firmly attached. A part of the surface f of block b is made conducting and in metallic connection with W; likewise a conducting part of surface f' is in metallic connection with W'. For holding the slab S a clamp operated by the screw s is used; for making the necessary adjustments in the position of S the nuts c, d, d' serve. The reason for making the surfaces f and f' partly conducting and in metallic



A, large electro magnet. *S*, experimental slab in position.
a, *a*, pole pieces of *A*. *B*, balance.

Fig. 1.

connection with W and W' respectively, will be made clear when the method used in making the field determination is described. In circuit with the magnet coils of A there is an ammeter, which with a variable resistance serves to keep the current constant during an experiment. In the same circuit there is also a commutator

which serves to change readily the direction of the current through the magnet coils.

A box, opened in front by means of a door, serves to protect the suspension apparatus from draughts.

An idea of the general distribution of the field is given in Fig. 3.

The slabs used in the experiments were rectangular in shape and of approximate dimensions 8 cm. by 4.5 cm. by .5 cm. When in

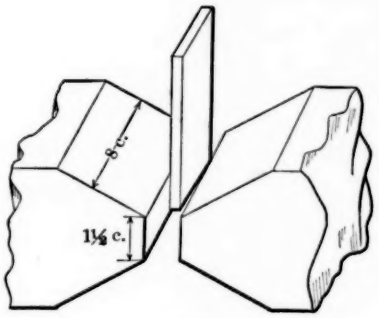
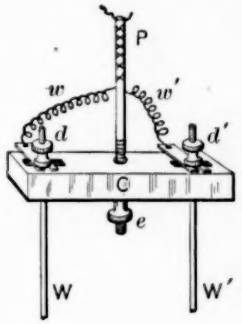


Fig. 2.



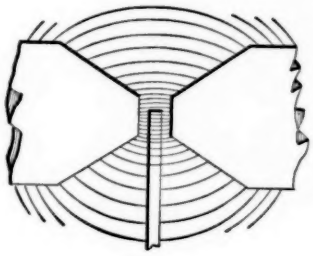


Fig. 3.

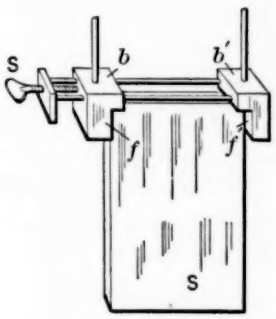


Fig. 4.

position for experimenting the long edges of the slab were vertical. Figs. 1, 2 and 3 show more clearly than could any description how the slabs were suspended.

Theory of the Method.

Considering magnetic forces due to stress in the media, it can then be shown [1] that the following equations must in general be satisfied for isotropic media.

[1] See, for instance, Webster, Electricity and Magnetism, p. 387.

$$X_x = \frac{1}{8\pi}(2\mathfrak{L}L - \mathfrak{B}H),$$

$$Y_y = \frac{1}{8\pi}(2\mathfrak{M}M - \mathfrak{B}H),$$

$$Z_z = \frac{1}{8\pi}(2\mathfrak{N}N - \mathfrak{B}H),$$

$$Y_z = Z_y = \frac{1}{4\pi}\mathfrak{M}N = \frac{1}{4\pi}\mathfrak{N}M,$$

$$Z_x = X_z = \frac{1}{4\pi}\mathfrak{N}L = \frac{1}{4\pi}\mathfrak{L}N,$$

$$X_y = Y_x = \frac{1}{4\pi}\mathfrak{L}M = \frac{1}{4\pi}\mathfrak{M}L,$$

where X_x represents the X-component of the force per unit of area upon a plane element perpendicular to the X-axis (an analogous meaning is to be given Y_y, Z_z and X_y, X_z, Y_x, Y_z, Z_x, Z_y), and L, M, N, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$, are the components of the magnetic force H and the magnetic induction $\mathfrak{B}$ respectively.

Referring to Fig. 3, let the axis of X be in the line of symmetry perpendicular to the pole faces; let the axis of Z be vertical and the axis of Y be perpendicular to the axes of X and Z. We wish to find an expression for the mechanical force acting upon the slab when placed relatively to the pole pieces as shown in Fig. 3. There will be discontinuity in the stress-components X_x, Y_y, Z_z, Y_z, Z_x, X_y at the surface of the slab where there is discontinuity in the permeability μ and the forces per unit area are equal to the amounts of the discontinuities. Let the permeability of the slab be μ_2 and let the permeability of the surrounding medium (air, in the present case,) be μ_1. The slab used is so thin that throughout its whole thickness, the tubes of induction may be assumed parallel to the X-axis, and consequently perpendicular to the faces perpendicular to the X-axis.

With this assumption we have then,

$$Z_z = Y_y = Y_z = Z_x = X_y = 0,$$

and $\iint X_x\, dydz$ taken over the two surfaces of the slab perpendicular to the X-axis equals zero. In a similar way the surface integral of the forces taken over the two surfaces perpendicular to the Y-axis

vanishes. But now consider the forces on the two faces perpendicular to the Z-axis, the conditions of symmetry applicable to the other faces do not hold here. Consider the lower Z-face, that is, the face between the poles. The equations show that all forces vanish at this face, save Z_z and we have then the force Z_z given in medium of permeability μ_1, by

$$Z_z = \frac{1}{8\pi} \mathfrak{B}_1 H_1,$$

and in the slab or medium of permeability μ_2 by

$$Z_z = \frac{1}{8\pi} \mathfrak{B}_2 H_2.$$

The difference in these two values gives us the total force per unit area acting upon the surface considered. If P_1 denote its value in grams-weight, we have

$$g P_1 = \frac{1}{8\pi} (\mathfrak{B}_2 H_2 - \mathfrak{B}_1 H_1)$$

But since H near the surface is the same in both media we have

$$g P_1 = \frac{1}{8\pi} (\mu_2 H_1^2 - \mu_1 H_1^2) = \mu_1 \frac{\varkappa_{21} H_1^2}{2}.$$

Since $\varkappa_{21}$, the relative susceptibility, is, by definition,

$$\varkappa_{21} = \frac{1}{4\pi} \left(\frac{\mu_2 - \mu_1}{\mu_1} \right).$$

Now a similar treatment of the forces acting at the top surface of the slab will give for the resultant force there,

$$g P' = \mu_1 \frac{\varkappa_{21} H'^2}{2}$$

where H' is the magnetic force at the top surface.

The difference between P_1 and P' times the area of the Z-face will give us the total force acting in the line of suspension. If A is the area of the Z-face, then P, the total vertical force, is given by

$$P = \mu_1 \frac{\varkappa_{21} A}{2g} (H_1^2 - H'^2).$$

Now experiment shows that H'^2 is negligible in comparison with H_1^2. If μ_1, the permeability of air, be assumed equal to unity, we

have $$P = \frac{\varkappa}{2g} AH_1^2$$

so that $$\varkappa = \frac{2gP}{AH^2}$$

where H is put for H_1 and $\varkappa$ for $\varkappa_{21}$.

For the determination of $\varkappa$ it is, accordingly, only necessary to determine P, the mechanical force acting on the slab, H the magnetic force at the bottom of the slab and the area of a cross section of the slab. In determining the strength of the field the balance mentioned above was used. A known current is sent through a conductor placed in that part of the field the strength of which we wish to measure, and then the vertical force F upon the conductor is determined by means of the balance. The field is given by the equation $$H = \frac{10gF}{Il}$$

where I is the current in ampères, l is the length of the wire in centimeters, F is the force in grams measured by the balance.

A rectangular slab of plaster-of-Paris was made with nearly the dimensions of the experimental slabs. On three edges of this slab (the bottom edge and the two side edges) were pasted thin strips of tinfoil just as wide as the slab was thick. Now, when this slab is clamped in the clamp of the suspension apparatus, so that the tinfoil of the side edges is in contact with the metallic part of the surfaces f and f', we shall have metallic connection between W and W', by way of the tin foil along the sides and across the bottom of the slab. When a field determination is to be made the slab is adjusted with reference to the poles in just the same manner as are the experimental slabs. Then the tinfoil strip across the bottom of the slab is a conductor in the field we wish to determine. Since the corners turned by the tin foil are very sharp the length of this conductor can be determined with great accuracy.

For the determination of the current I a Weston millivolt-meter was used in connection with a German-silver shunt. The millivolt-meter was carefully calibrated by means of a Rayleigh current-weigher, constructed by Mr. S. N. Taylor, in connection with his work on Clark and Weston cells.

Method of Experimentation.

The substance for a series of experiments having been selected, the field magnets were excited with the proper current regulated by a suitable resistance. An assistant kept the needle of an ammeter in circuit with the magnet coils, always over a selected line of the scale. The plaster-of-Paris slab, described above, was then introduced into the proper position. Equilibrium was obtained when no current passed through the conductor in the field and a reading of the balance taken. A current measured by the Weston millivoltmeter was then sent through the conductor and a new reading taken when equilibrium had been again secured. The difference in these two readings gave the force in grams F due to the field, upon the current in the conductor. Then the plaster slab was removed and a slab of one of the substances to be investigated introduced and a reading of the balance taken when equilibrium obtained. The slab was then reversed, top for bottom, and another reading obtained. This process was then gone through with all the five or six substances selected for a particular series of experiments, another determination of the field being made in the middle of the series and still another at the end.

After the last field determination, and while the field magnets were still excited, a reading of the balance was obtained with no slab in the clamp of the suspension apparatus. This reading was to enable us to make correction of any magnetic action upon the suspension apparatus itself. The current through the magnet coils was then cut off. The wires w and w' were hung in long loops from fixed supports in the balance case to a little rubber support firmly attached to the suspension rod P. From this, being well insulated, they were twined about the rod P and connected with W and W' at the nuts d and d' as shown in Fig. 4. These wires would vary slightly in length when a current passed and correction had to be made for this. Both wires w and w' were connected with one nut (d, Fig. 4) and the same current used in making the field determination was thus sent over the flexible Cires in the balance case, without going near the field magnets. A reading of the balance was taken when the current had been properly adjusted.

The residual field was then eliminated with great care. This was done by reversing over and over again a small current through the magnet coils until the balance gave no indication of a field, when a current was sent through the conductor arranged as when the field was determined.

The slabs were then, each in turn, clamped in the clamp of the suspension apparatus and readings of the balance with the zero field taken. These readings, with those taken with the field on and with the proper corrections mentioned above, are sufficient to determine the mechanical force in the Z-direction. This force we have, in the theory of the method, called P.

A scale graduated to read fifths of a millimeter, was used to determine the width of the slab and a Brown and Sharpe micrometer screw gauge graduated to read thousandths of a centimeter, was used to determine the thickness of the slab. The product of this width and thickness gave the area A. In making these width and thickness measurements, many measurements were taken on a given slab and the mean of these used in the calculations.

Results.

The tabular results are, in general, arranged in groups which give the substances experimented upon and the data obtained in a given series of experiments.

A table of abbreviations which will be found convenient in referring to the tables of data is given below:

H. Unit tubes of force per square centimeter at bottom surface of slab.

p_1. Balance reading in grams, field on and slab in position.

p_0. Balance reading in grams, field off and slab in position.

p_1'. Balance reading in grams, field on, plaster slab in position and current through conductor.

p_0'. Balance reading in grams, field on and plaster slab in position but no current through conductor.

P. Mechanical force, in grams, upon the slab.

F. Mechanical force, in grams, upon conductor in field when current is on.

I. Current in ampères through conductor when field is being measured.

ε. Correction necessary on account of magnetic attraction upon suspension apparatus without slab.

∂. Correction necessary on account of extension in wires w and w' when current is sent through them.

l. Length in centimeters, of conductor used in measuring field.

w. Mean width in centimeters, of experimental slab.

t. Mean thickness, in centimeters, of experimental slab.

$\varkappa$. Coefficient of magnetic susceptibility.

In the first series of experiments, determinations of $\varkappa$ were made for five samples of marble. In the first column of the table are found the names of the different varieties of marble experimented upon. The second column gives p_1 two values being given for each substance, since each slab is reversed in each experiment, the top being put for the bottom; and the third column gives the mean of these two values. The fourth column gives p_0 the reading of the balance when slab is in position, but no field on. In the fifth column are found the values of $P = p_1 - p_0 + \varepsilon$. In the next two columns are found w and t, the factors of A, the area of the slab. The last column gives the values of $\varkappa$ multiplied by 10 to the sixth power, which values are calculated from the data found in the columns and below. Below the columns are found the corrections ε and ∂, the necessary data for the calculation of H, and the calculated value of H. The same value of H was used throughout a given series of experiments. The same arrangement of data and results is followed in all of the tables given.

Special attention was thus bestowed upon the study of the magnetic properties of marble, since it has been often used as the means of support for coils in standard apparatus. The results obtained seem to indicate that almost any variety of marble can be relied upon to be diamagnetic and free from iron and to have a magnetic susceptibility of about $-.8 \times 10^{-6}$. Therefore, it would seem that a free use of marble in magnetic apparatus is allowable.

Table II. gives the data and results for aluminium, glass and tin.

It should be noticed that the coefficient of susceptibility for tin is positive, that is, the present experiments would seem to indicate that

TABLE I.—MARBLE.

	p_1	Mean p_1	p_0	$P = p_0 - p_1 + \epsilon$	w	t	$\kappa \times 10^6$
Italian, . . .	103.3462 103.3463	103.3462	103.4244	.0792	5.18	.490	—.940
Italian Statuary (1), .	101.2129 101.2146	101.2138	101.2798	.0670	5.13	.474	—.832
Rutland Statuary, . .	102.7416 102.7382	102.7399	102.8070	.0681	5.14	.502	—.795
Gray Knoxville, . .	103.7836 103.7832	103.7834	103.8336	.0512	5.12	.500	—.603
Italian Statuary (2), .	102.8600 102.8620	102.8610	102.9279	.0679	5.14	.491	—.811

p_1'	p_0'	$F = p_1' - p_0' + \delta$	H
74.6630	73.0726	1.593	8.060

$\epsilon = .0010$ $l = 4.54$
$\delta = .0027$ $I = .427$

TABLE II.

	p_1	Mean p_1	p_0	$P = p_0 - p_1 + \epsilon$	w	t	$\kappa \times 10^6$
Aluminium,	111.5490 111.5565	111.5527	111.3680	−.1837	4.49	.650	+1.88
Glass, . . .	92.5120 92.5120	92.5120	92.5524	+.0414	3.97	.536	− .578
Tin, . . .	162.0290 162.0295	162.0293	162.0048	−.0235	4.52	.436	+ .354

p_1'	p_0'	$F = p_1' - p_0' + \delta$	H
74.5290	72.9274	1.604	8.120

$\epsilon = .0010$ $l = 4.54$
$\delta = .0024$ $I = .427$

tin is magnetic. Now, previous experiments upon tin have clearly shown it to be diamagnetic, and the conclusion to be drawn in the present case is that the sample used must have contained traces of iron. The glass used was a piece of fine optical glass. The value of $\varkappa$ obtained for this piece of glass is four or five times the value usually given. Aluminium is seen to have a positive value of $\varkappa$ and quite large.

TABLE III.

	p_1	Mean p_1	p_0	$P = p_0 - p_1 + \epsilon$	w	t	$\kappa \times 10^6$
Antimony,	173.3254 173.3320	173.3287	173.3830	.0553	4.55	.495	− .714
Bismuth,	202.5290 202.5970	202.5630	203.3932	.8312	4.58	.430	−12.25
Sulphur,	96.3017 96.2963	96.2990	96.3817	.0837	4.94	.644	− .765
Ebonite,	80.9105 80.9232	80.9168	80.7964	.1194	4.94	.648	+1.08
Paraffin,	64.4940 64.5054	64.4997	64.5480	.0493	4.44	.559	− .577
White Wax,	63.7308 63.7298	63.7303	63.7811	.0518	4.94	.544	− .560
Shellac,	74.1848 74.2178	74.2013	74.2422	.0419	5.03	.615	− .394

p_1'	p_0'	$F = p_1' - p_0' + \delta$	H
74.6347	73.0140	1.623	6.740

$\epsilon = .0010$ $l = 4.54$

$\delta = .0026$ $I = .427$

The bismuth used in this series was furnished by Eimer and Amend, of New York, and was supposed to be chemically pure. The results for antimony seem to show that the sample used must have contained just a trace of iron, since the value found for κ while negative is much smaller than that usually given.

Table IV. gives the results found for several kinds of wood.

These results show, in general, that the greater the density of the wood the greater is the coefficient of magnetic susceptibility.

Besides these quantitative experiments there were made a large number of qualitative experiments upon a great many different kinds of wood. Only one kind out of about twenty different kinds tried proved to be magnetic, the rest were diamagnetic; the exception was butternut.

There has been in the past a difference of opinion among physicists as to whether diamagnetic substances are or are not less susceptible under the application of large magnetizing forces. Most of the earlier experimenters, among them Tyndall, Joule and Becquerel, found reason to believe that κ is constant for diamagnetic substances.

TABLE IV.

	p_1	Mean p_1	p_0	$P = p_0 - p_1 + \epsilon$	w	t	$\kappa \times 10^6$
White Wood,	57.5927 57.5934	57.5930	57.5957	.0029	5.09	.462	−.18
Holly, . .	60.9816 60.9812	60.9814	60.9857	.0045	5.07	.499	−.26
Plain Oak, .	61.6704 61.6704	61.6704	61.6766	.0064	5.04	.449	−.36
Tulip, . . .	69.0779 69.0779	69.0779	69.0863	.0086	5.08	.476	−.51
Cedar . .	58.1218 58.1210	58.1214	58.1239	.0027	5.03	.479	−.16

p_1'	p_0'	$F = p_1' - p_0' + \delta$	H
73.6900	72.9620	.7314	3.700

$\epsilon = .0002$ $l = 4.54$

$\delta = .0034$ $I = .427$

TABLE V.—BISMUTH.

	p_1	Mean p_1	p_0	$P = p_0 - p_1 + \epsilon$	w	t	$\kappa \times 10^6$
Bismuth (1)	203.0078 203.0090	203.0084	203.0414	.0330	4.58	.430	−12.55
Bismuth (2),	202.8664 202.8810	202.8737	203.0400	.1663	"	"	−12.25
Bismuth (3),	202.5290 202.5970	202.5630	203.3932	.8302	"	"	−12.25
Bismuth (4),	202.0209 202.1010	202.0959	203.0400	.9441	"	"	−12.55
Bismuth (5),	201.6156 201.7310	201.6833	203.0400	1.367	"	"	−12.45

Plücker, however, believed that in some cases as the magnetizing force is increased the coefficient of susceptibility for diamagnetic bodies decreases in a similar way as in iron. Plücker also thought he observed in some cases an increase in $\varkappa$ with an increase in the magnetizing force up to a certain point, after that a decrease; that is, for a certain value of the magnetizing force $\varkappa$ has a maximum.[1]

[1] Wied. Ann. II., p. 324. 1880.

Silow (1880) published results which indicate that $\varkappa$ for ferric chloride increases for a while with the magnetizing force, reaches a maximum and decreases.

The fifth series of experiments was undertaken in the hope of showing definitely whether bismuth, the strongest diamagnetic substance known, does or does not vary in its susceptibility to magnetization as the magnetizing force is made to vary.

Table V., which gives the results of experiments upon bismuth, using widely different fields, is arranged in a way similar in all respects to the preceding tables.

	ϕ_1'	ϕ_0'	$F = \phi_1' - \phi_0' + \delta$	H
(1)	72.9950	72.6986	.2984	1,620
(2)	73.3670	72.6914	.6764	3,680
(3)	74.6347	73.0140	1.623	8,210
(4)	74.2720	72.6514	1.622	8,800
(5)	74.5500	72.6270	1.925	10,450

Note—The current used in making the field determination was in all cases except the third .398 ampères; in the third case it was .427 ampères, $l = 4.54$. δ was approximately equal in all cases to .0020. ε was negligible.

As the table shows, the field used in the last series of experiments vary between the limits of 1,610 and 10,450 lines per square centimeter. The results show that within these wide limits $\varkappa$ is constant for bismuth.

A similar series of experiments upon white wax indicated that for this substance $\varkappa$ is constant within wide limits of the field strengths; and in the light of these results it would seem probable that for solids, at any rate, $\varkappa$ is constant.

The method for the study of magnetic and diamagnetic phenomena in solid bodies, described in the present paper, is easily adapted to the study of liquids, and of gases under different pressures.

At various intervals during the present century many different physicists have been interested in the study of diamagnetism. But, notwithstanding the fact that many have worked upon the problems which a study of diamagnetism offers, there seem to be yet some things in connection with the subject which merit more careful investigation. For instance, the question as to whether or not a dia-

magnetic body, after being under the influence of a magnetizing force and this force withdrawn, requires, if left mechanically undisturbed, a "coercive force" to bring it to the neutral magnetic state again, is still an open one. Of course, the extreme smallness of the action to be measured is the great difficulty encountered in any quantitative work in diamagnetism.

For the illustrations used in this paper I have to thank Mr. Joseph Daniels, of the Colorado State Normal School. I am under great obligations to Dr. A. G. Webster, Professor of Physics at Clark University, for suggestions and advice given while the work was being carried out; also to Clark University for providing the necessary apparatus.

MINOR CONTRIBUTION.

A Résumé of the Experiments dealing with the Properties of Becquerel Rays.

By Oscar M. Stewart.

THE literature on Becquerel rays is quite scattered, consisting of a large number of articles, only a few of which are in English. Knowing this and believing the subject deserving of more attention than it has received, it has been thought advisable to give here a summary of the present state of our knowledge of the subject, together with its bearing on the subject of X-rays.[1]

The discovery of X-rays and the use of fluorescent screens with them aroused a greater interest in fluorescence and phosphorescence. This increased interest and the use of fluorescent and phosphorescent screens with photographic plates led to the discovery that some of these chemicals emitted a sort of invisible radiation. An effect from several different chemicals was noticed, but the radiation from uranium and its salts differs from the rest, as will presently be seen. Becquerel rays, or uranium rays, as Becquerel himself called them, are the rays given off from metallic uranium or any of its salts. This radiation at first appears to be a phosphorescence. Sylvanus Thompson[2] has suggested the name of "hyperphosphorescence" for it and allied phenomena.

Becquerel rays occupy a unique position, inasmuch as far more is definitely known about them than any of the other 'new' "rays." With X-rays nothing has been proven one way or the other about their character, save that if they are ultra-violet rays their wave-length must be extremely small, so small that the refractive index for nearly all bodies is practically unity. With the rays of Becquerel there can be no reasonable doubt that they are short transverse ether waves.

A great deal of the work that has been done with the radiation from fluorescent salts has been done with the aid of photography. It is well to state at the outset that, as is now rather well known, photographic results are often misleading. Many cases where a radiation was thought

[1] For earlier abstracts see Sagnac, Journal de Physique, p. 193, 1896. Also, S. P. Thompson, "Light, Visible and Invisible," p. 278.

[2] Philosophical Magazine, s. 5, 42, p. 103.

to exist can be explained by a chemical effect, either direct or resulting from the vapors given off by the salt when laid on or near the photographic plate. Much interest was excited but a short time ago over a supposed invisible radiation from glow worms. The discoverer[1] has lately announced that the effect was in some way due to moisture, it being necessary to keep the glow worms wet. Moistened paper gave the same effect that the glow worms had. In this last article the authors call attention to some similar but anomalous phenomena of pseudo-radiation from cadmium oxide, which can not be entirely explained by the chemical action of a vapor. It has been announced in several places that freshly cleaned zinc gives a radiation which will affect photographic plates. M. Colson[2] has shown that magnesium and cadmium give the same effect, and that the action is due to a vapor given off by the freshly cleaned surfaces. In some cases cited in this paper where a radiation from some chemical is said to have been found, no detailed statement has been made of the experiment so that it is not possible to draw any conclusion in regard to their reliability.

In the early part of February, 1896, Charles Henry[3] published the statement that sulphide of zinc augmented the photographic effect of light and X-rays. He claimed to have found that if he covered an object with this sulphide the photographic effect of X-rays was increased, showing that bodies partially opaque to X-rays became less so when covered with this zinc sulphide. This he regarded as a "new proof of the complexity of the radiations emitted from a Crookes' tube." He found that this sulphide when exposed to magnesium light or daylight acted photographically through black cardboard and aluminum. For diffused daylight the exposure was from three to five hours. A week later Niewenglowski[4] published the statement that commercial sulphide of calcium emitted radiations which traversed opaque bodies. Becquerel,[5] two weeks later than Henry, announced that this statement might be extended to other phosphorescent bodies, in particular to the salts of uranium, in which the phosphorescence is very short. He wrapped a photographic plate with two thicknesses of heavy black paper such that the plate was not fogged by an exposure to the sun for a day. He placed on the sheet of paper the phosphorescent material, double sulphate of uranium and potassium, and exposed it to sunlight for some hours. On developing the plate a silhouette of the substance appeared in black on the negative. When a coin or any metallic screen was interposed, shadows of these appeared on the plate. The effect was still obtained when a piece of glass was interposed, which prevented a direct chemical action.

[1] Muraoka and Kasuya, Wiedmann's Annalen, 64, p. 186, 1898.
[2] Comptes Rendus 123, p. 49. See also J. J. Thomson, Camb. Phil. Soc., 9, p. 372.
[3] C. R., 122, p. 312. [4] C. R., 122, p. 385. [5] C. R., 122, p. 420.

In his next article[1] he states more about this compound. The visible phosphorescence is very bright, but its duration is less than one one-hundredth of a second. Continuing the experiments first described he placed a plate in a frame wrapped in black cloth and closed on one side by a plate of aluminum. When exposed to full sunlight for a day the plate was not fogged, but when some of the uranium salt was placed on the aluminum plate and exposed to the sun for some hours, the photographic action was marked. "If the aluminum plate was a little thick, the action was less than across two sheets of black paper." Between the salt and the aluminum plate was placed a screen of copper, 0.1 mm. thick, in the form of a cross. This gave a distinct shadow, but showed that the copper plate was partially transparent.

One day he had prepared several plates in this way, with a screen and some of the crystals on it, but as the day was cloudy and the sun appeared only occasionally, he put the plates away in a drawer all prepared. Unfortunately, as he then thought, it remained cloudy. After waiting for several days he developed the plates, expecting to find perhaps a feeble image, but was surprised to find that the images appeared with great intensity. He found in this accidental way that it was not necessary to expose the crystals to sunlight or daylight. Crystals laid on plates affected the plates when they were left in a dark room. It must be remembered that these phenomena could not be attributed to the visible phosphorescence, as the latter lasted only about 0.01 sec.

Becquerel at first said that he thought that these invisible radiations were due to a phosphorescence, the persistence of which was much greater than the duration of the visible phosphorescence. If the radiation is a phosphorescence, it ought to be stimulated by exposure to light. This was found[2] to be true for daylight and the light from an arc lamp, but magnesium light and X-rays[3] produced no increase. On the other hand he obtained[4] radiations from uranium salts which were neither fluorescent nor phosphorescent so far as is known.

Nitrate of uranium ceases to phosphoresce or fluoresce when dissolved. To further test the relation between these invisible radiations and the phosphorescence, Becquerel[5] took a crystal of this salt and placed it in a tube. It was warmed in the dark, being carefully protected from the radiation from the alcohol lamp. After the salt had dissolved, he permitted it to crystallize, still keeping it in darkness. When placed on a photographic plate covered with black paper it acted strongly on the plate. The solution of uranium nitrate[6] although not fluorescent, also emitted these invisible radiations.

[1] C. R., 122, p. 501.
[2] C. R., 122, p. 691.
[3] C. R., 122, p. 766.
[4] C. R., 122, p. 691.
[5] C. R., 122, p. 691.
[6] C. R., 122, p. 765.

As the rays were given off by the salts of uranium whether they were fluorescent or not, he was led to the conclusion that it must be a property of the metal. He found,[1] on testing, that metallic uranium gave the effect with about four times the intensity obtained from any of the salts he had used.[2]

Although this seems to be a phenomenon of invisible phosphorescence, it is evidently not intimately connected with visible fluorescence or phosphorescence. Further, if it is a phosphorescence, uranium is then the only metal known to give it.

Using the double sulphate of uranium and potassium and metallic uranium, Miethe[3] confirmed Becquerel's observations showing that the rays would transverse aluminum.

Duration of the Radiation.

When Becquerel discovered that the uranium salts did not need to be exposed to light to give off this radiation, he put some samples away in the dark and tested them from time to time. The results were announced in the Comptes Rendus in his various articles. But it is only the later ones that interest us. Salts[4] which had been kept in darkness for six and some for eight months showed no sensible diminution of their emissive properties. Some of these salts had been sealed in glass tubes and kept in boxes of lead, where all known radiation, except from the sides of the box, was excluded. Still later[5] he announced that the salts which he had kept in darkness for more than a year, all known radiation being excluded, continued to give these rays with an intensity scarcely decreasing (*à peine décroissante*).

The emission of some of the salts[6] which had been kept in obscurity for two months, when exposed to the sun, or better, to the light from an arc lamp or electric spark, was slightly increased, but in a few hours it was back to its normal state.

The source of the energy radiated from uranium kept in darkness is at present unknown.

Elster and Geitel[7] have confirmed Becquerel's work on the duration of the emission of the radiation from uranium.

[1] C. R., 122, p. 1086.

[2] These comparisons were made with an electroscope. The method is here described later. The uranium used was chemically pure having been prepared by M. Moissan.

[3] Beiblätter, Wied. Ann., p. 606, 1897.

[4] C. R., 123, p. 856.

[5] C. R., 124, p. 803.

[6] C. R., 122, p. 1086.

[7] 10 Jaresber. Ver. Naturw. Braunschweig, 1897. Beibl., p. 455, 1897.

Reflection and Refraction.

To test the character of these rays Becquerel began a series of experiments which were quite successful. He was eventually able to show reflection, refraction, and polarization. Owing to the extreme feebleness of these rays it was entirely out of the question to show diffraction.

On a plate[1] was placed a layer of the double sulphate of uranium and potassium, and placed over the top of half of this was a polished steel mirror with its face down. The plate, when developed after an exposure of fifty-five hours, showed a difference between the two halves indicating that the steel mirror had reflected the rays. A hemispherical mirror was polished from a block of tin and regular reflection was obtained with exposures of forty-six hours.

Owing to the feeble intensity of the radiation, he was at first unable to detect refraction, but later experiments[2] were successful. On one face of a prism of crown glass was fastened, parallel to the refracting edge, a glass tube of about 1 mm. diameter. This tube was filled with nitrate of uranium and formed a linear source. The prism was laid with the other face on a photographic plate. After an exposure of three days the plate showed a refraction of about the same magnitude as that obtained with ordinary light. No determination could be made of the refractive index of the prism for these rays.

To protect it from moisture some nitrate of uranium had been put[3] in a bell-shaped glass tube one end of which had been sealed with paraffin to a thin glass plate. This was placed in an upright position on a photographic plate. The negative showed plainly that the rays had been refracted and totally reflected in the glass tube in the same way that light follows a liquid vein of water. This effect was also obtained from calcium sulphide. Becquerel said: "If the phenomena of reflection and refraction had not been shown in any other way it was clearly shown by this proof alone."

The fact that pulverized glass seemed transparent[4] was apparently contradictory. In fact, the photographic plate was affected more under the pulverized glass than elsewhere. Becquerel made no attempt, so far as we are aware, to explain this anomaly. If this particular quality of glass with respect to air had a refractive index of about unity for Becquerel rays, when pulverized it should appear transparent. It may be noted that pulverized glass is transparent for X-rays.

Polarization.

Becquerel[5] found that the rays could be polarized and that tourmalin showed dichroism. A piece of tourmalin, 0.5 mm. thick and with its faces

[1] C. R., 122, p. 561.
[2] C. R., 122, p. 693.
[3] C. R., 122, p. 562.
[4] C. R., 122, p. 766.
[5] C. R., 122, p. 762.

parallel to the optic axis, was cut in two, and the two pieces were laid on a photographic plate with their optic axes at right angles. A third piece of tourmalin, 0.88 mm. thick and also cut parallel to the optic axis, was laid on top of the first two in such a way that its axis was parallel to the axis of one of the pieces and perpendicular to that of the other. The photographic plate showed clearly the existence of dichroism, that is, the crossed tourmalins were opaque, while the part where the axes were parallel was transparent. This experiment showed double refraction, polarization, and the dichroism of tourmalin.

With X-rays the same tourmalins gave a negative result. This did not prove that the X-rays could not be polarized: it only showed that there was no unequal absorption of the two rays by tourmalin.

Discharge of an Electrified Body.

Becquerel[1] found that, like X-rays and ultra-violet light, these radiations would discharge an electrically charged body. He used the electroscope of M. Hurmuzescu, which was surrounded by a metallic sheath for guarding against electrostatic action and had windows of yellow glass to keep out ultra-violet light. This electroscope would hold its charge for a long time, a month it is said. He replaced one of the yellow glass windows with aluminum, against the outside of which was placed the uranium salt. A slow discharge was then detected. A plate of the double sulphate of uranium and ammonium, 45 mm. by 25 mm., which had been kept in the darkness for five days, was placed under the gold leaves, the latter having been given a charge sufficient to cause a divergence of 12°. By watching the rate of discharge quantitative results could be obtained. Under these conditions it took from 21 to 25 minutes to completely discharge either a positive or negative charge.

Absorption.

Becquerel[2] found that he could use the electroscope of Hurmuzescu to determine the absorption of different substances. Double sulphate of uranium and potassium dissipated a charge with a velocity 4.15 times as fast as when a plate of quartz, 5 mm. thick and cut perpendicular to the optic axis, was interposed. X-rays gave the ratio 15.7, showing that quartz was more transparent to Becquerel rays.

Two methods were thus available for testing absorption, namely, photography and the electrical discharging action. It was found by Becquerel[3] that the following substances were rather transparent to the radiation from the salts of uranium: water; most solutions, even of metallic salts

[1] C. R., 122, p. 559.
[2] C. R., 122, p. 689.
[3] C. R. 122, p. 692 and 763.

such as copper nitrate, chloride of gold, and nitrate of uranium; an alcoholic solution of chlorophyl; paraffin; quartz; Iceland spar; and sulphur. Quartz is more opaque than Iceland spar. Uranium glass, a red glass, and a blue cobalt glass were more opaque. Copper was nearly as transparent as aluminum. Platinum of about one-third the thickness of a sheet of aluminum was about as transparent. Air[1] was found to slightly absorb the radiation.

The absorption of these rays was compared[2] to the absorption of X-rays and he found that most substances, particularly the metals, were more transparent to uranium rays than to X-rays.

Becquerel[3] found with the electroscope that the absorption of these rays by two metals, platinum and aluminum, superimposed was less than the sum of the effects taken separately. This shows that the rays are not homogeneous. The heterogeneity of X-rays has been shown in the same way.

Conductivity of Gases.

In November, 1896, Becquerel[4] added another important fact to our knowledge of uranium rays, viz., that they, like X-rays, impart to gases the property of discharging electrified bodies. His experiments were made with air and carbon dioxide. As has been already stated he had found that these rays discharged an electrified body. But now he found, as had been found with X-rays, that the rays seem to make the gas through which they pass a conductor and that this gas after being acted upon by the rays from metallic uranium would discharge an electrically charged body when brought in contact with it. Rays from the double sulphate of uranium and potassium gave the same result.

Becquerel[5] found that metallic uranium, although conventionally insulated, would not hold a charge unless it was kept in a vacuum. He made some measurements to find the dependence of the rate of discharge of the uranium on the density of the surrounding gas. It was found to be for potentials of 15 volts proportional to the square root of the density. This should not be accepted as final, as the law probably depends a great deal on the potential. This is the law which Benoist and Hermuzescu[6] proposed for the case of X-rays. But wide deviations from this law have since been found at lower voltages.[7]

The analogy of the discharge of electricity from uranium to the discharge of negative electricity from metals by ultra-violet light is at once suggested. Elster and Geitel[8] tested zinc, aluminum, and other substances

[1] C. R., 122, p. 693.
[2] C. R., 122, p. 763.
[3] C. R., 122, p. 765.
[4] C. R., 123, p. 857.
[5] C. R., 124, p. 438.
[6] C. R., 122, p. 926 and 123, p. 1265.
[7] See C. D. Child, PHYSICAL REVIEW, 5, p. 193 and 285.
[8] 10 Jaresber. Ver. Naturw. Braunschweig, 1897. Beibl., p. 455, 1897.

which present photo-electric effects, *i. e.*, a discharge of negative electricity by ultra-violet light, and found that they gave no radiation similar to Becquerel rays. Hence they conclude that photo-electric phenomena cannot be explained by a hyper-phosphorescence of the charged metals.

Becquerel's last two papers deal chiefly with the dependence of the rate of discharge on the potential, the density being kept constant. For low potentials[1] the law of discharge is identical with the law of cooling, that is the rate of fall of potential, $\frac{dV}{dt}$, is proportional to the potential V. But for larger potentials the value of $\frac{dV}{dt}$ increases very slowly with potential and tends toward a constant. This he[2] finds can be expressed by the equation $\frac{dV}{dt}\left(a + \frac{b}{V}\right) = -1$

where a and b are constants and proportional to the capacity of the discharging bodies. In different experiments a varied from 4.1 to 4.5 and b from 20 to 31.

It will be noticed that, although the gas seems to act as a conductor, for higher potentials it does not obey Ohm's law. So far as any comparison can be made, all these results agree with the results which have been obtained with X-rays.

Beattie and de Smolan[3] tested the rate of discharge of electricity by uranium and compared it with the discharge by ultra-violet light and X-rays. They also tested the rate of discharge from uranium in different gases and at different pressures. They found that the discharge by uranium, as with X-rays, was not proportional to the potential. Their results for this agree with those of Becquerel. They, like Becquerel, found that positive and negative charges were dissipated at the same rate and that the rate of discharge by uranium was not perceptibly increased when it was heated or exposed to the sun. It may be stated in this connection that they found that ultra-violet light discharged positive and negative charges at the same rate, "the positive or negative charge being reckoned from the steady electrometer reading which is obtained when the two quadrants of the electrometer are insulated and the ultra-violet light shining." This result agrees with that obtained by Branly, but it has been contradicted by others.[4]

They tested the rate of discharge for different potentials of an insulated piece of uranium in hydrogen, oxygen, carbon dioxide as well as in

[1] C. R., 124, p. 444.

[2] C. R., 124, p. 800.

[3] Phil. Mag., s. 5, 43, p. 418. Proc. Roy. Soc. Edin., 21. p. 393.

[4] See article by Merritt, "The Influence of Light upon the Discharge of Electrified Bodies," Science, 4, pp. 853 and 890.

air. For all potentials the rate in air was less than in oxygen, and in hydrogen it was less than in air. The rate of discharge in carbon dioxide is greater than in hydrogen, but as its rate increases more rapidly with potential than air and oxygen, for high potentials the rate becomes larger than that for air or oxygen. In air for potentials of four and ninety-six volts the rate of discharge was nearly proportional to the atmospheric pressure. With hydrogen the rate of discharge was "at higher pressures somewhat approximately proportional to the pressure, at lower ones to the square root of the pressure." With oxygen the rate was approximately proportional to the square root of the pressure.

J. C. Beattie[1] showed that the gas itself if near an electrified body became electrified when exposed to uranium rays. He placed an insulated piece of uranium inside of a metallic cylinder. The air from around the uranium could be drawn through an electric filter, an insulated tube of block tin filled with brass filings connected to an electrometer. When the cylinder and uranium were both grounded, the air acquired a slight positive charge. When the uranium was kept positively charged the air became positive even when the uranium was wrapped in aluminum foil or surrounded by paraffin and tin foil. When negatively charged the air became negative. The results clearly indicated a slight difference between the amount of negative and positive electricity given to the air, the positive being the larger, but the difference was of about the same magnitude as the charge obtained when both the cylinder and uranium were grounded. Uranium acetate and uranium nitrate were tested with the same results although the action was not so great. No increase in the electrification of the air was observed when ultra-violet light was allowed to fall on the acetate. When a piece of lead was substituted for the uranium no effect was obtained.

It had been shown[2] that when X-rays passed between two insulated metals, these metals took on final charges equal to those they would have if connected by a drop of water. Beattie and de Smolan[3] found the same effect with Becquerel rays. Righi in his earlier experiments found a similar effect with ultra-violet light. However, Beattie and de Smolan[4] continued the study of this effect, using X-rays, ultra-violet light, and Becquerel rays. They found as Righi had in his later work that the potential difference of two metallic plates when exposed to ultra-violet light depended upon the distance between the plates. Hence the potential difference found could not be the contact electromotive force. With

[1] Phil. Mag., s. 5, 44, p. 102. Proc. Roy. Soc. Edin., 21, p. 466.

[2] Murray, Proc. Roy. Soc. Lond., 59, p. 333. Perrin, C. R., 124, p. 496. Kelvin, Beattie and de Smolan, Nature, 55, p. 344.

[3] Phil. Mag., s. 5, 45, p. 277. Nature, 55, p. 447.

[4] Phil. Mag., s. 5, 43, p. 418. Nature, 56, p. 20. Proc. Roy. Soc. Edin. 21, p. 393.

X-rays and rays from uranium no change was detected when the distance between the plates was varied. The potential difference between two metals depended on the nature of the surface of the metals, that is, whether polished or not.

The difference of potential between two plates due to uranium rays when gases at different pressures were between the plates was tried by them. For hydrogen, oxygen, and air the potential difference at different pressures was approximately the same. They state that we are not warranted in saying that the difference of potential developed by X-rays and Becquerel rays is the volta difference between the electrically effective surfaces of the two metals. In the case of Becquerel rays the potential difference observed was in the same direction but larger than the potential difference obtained when the plates were connected by a drop of water.

The various phenomena connected with the action of X-rays on gases can be explained on the hypothesis given by J. J. Thomson and Rutherford[1] and the same explanation holds for Becquerel rays. This theory assumes that the gas is in some way broken up into ions which can carry electric charges. In other words the gas becomes a sort of electrolyte.

Formation of Fog.

It has been shown by Wilson[2] and Richardz[3] that X-rays will cause a condensation of water in supersaturated dust-free air. Wilson[4] has shown that the rays from the double sulphate of uranium and potassium produce the same effect. Sometimes only a light fog can be seen, while at other times a dense shower is visible. The effect of the gas will persist for some seconds after the radiation has been cut off. The same thing has been observed with ultra-violet light, but Lenard and Wolf found that there was no fog formation unless the action takes place in the neighborhood of a charged body so that there may be fine particles driven off from it to form the nuclei for the drops of water. Wilson, following the explanation of Thomson, thought that the ions in the dissociated gas form the nuclei for the formation of the drops of water.

Thermo-luminescence.

Hoffman[5] found that Wiedemann's discharge rays would excite thermo-luminescence, that is, certain substances after an exposure to these rays would become luminous when heated. Borgman[6] found that this thermo-

[1] Phil. Mag., S. 5, 42, p. 392.
[2] Proc. Roy. Soc., 59, p. 338. Phil. Trans., p. 265, A. 1897.
[3] Wied. Ann., 59, p. 592.
[4] Proc. Phil. Soc. Camb., 9, p. 333.
[5] Wied. Ann., 60, p. 269, 1897.
[6] C. R., 124, p. 895.

luminescence was excited also by X-rays and Becquerel rays. He used a mixture of calcium sulphate and manganese sulphate. This mixture gave no phosphoresence or thermo-luminescence after a long exposure to an arc light. But the effect was very marked under the action of the discharge rays coming from a spark of a Voss machine. For Becquerel rays it took long exposures. When exposed for six days the action was quite strong.

Excitation of Fluorescence.

P. Spies[1] found that with the rays of Becquerel he could produce no visible fluorescence, but that a fluorescence could be detected photographically. A sheet of bromide of silver paper was laid on a piece of fluorspar. This was then wrapped in black paper and exposed to the radiation from uranium. The paper next to the fluorspar was very strongly blackened. A lead plate placed between the bromide of silver paper and the uranium would act as a screen.

F. Maack[2] found that uranium rays which had gone through pulverized resin (*Kolophonium*) acted more strongly on a photographic plate than when the resin was not present. He thought that the uranium rays were changed into less refrangible but more actinic rays. The latter he found to penetrate cardboard with more difficulty than the former.

Radiation from Other Salts.

As has been stated Henry found that zinc sulphide, and Niewenglowski found that sulphide of calcium, emitted rays which traversed opaque bodies. Troost[3] also published a note saying that he had obtained shadowgraphs of metallic objects with a radiation from hexagonal blende (zinc sulphide) rendered phosphorescent by magnesium light, and recommended its use in the place of Crookes' tubes. Arnold[4] found that Becquerel rays are emitted by zinc sulphide, fluorspar, mixtures of various sulphides, and retene, but that many strongly luminescent solid solutions send out no Becquerel rays. Steffani[5] found that certain phosphorescent substances give rays of short wave-length which pass through opaque substances.

From a sulphide of calcium that luminesces blue and from another that luminesces a greenish blue Becquerel[6] obtained through glass and 2 mm. of aluminum the strongest effects he had obtained up to that time. The

[1] Verhand. d. Physikal. Ges., Berlin, **15**, p. 101, 1896. Beibl., p. 67, 1897.
[2] Metaphys. Rundsch., '96–'97. Beibl., p. 366, 1897.
[3] C. R., **122**, p. 564.
[4] Wied. Ann., **61**, p. 313. Phys. Soc. Lond., Abst., p. 320, 1897.
[5] Beibl., p. 916, 1896.
[6] C. R., **122**, p. 563.

rays were also shown to be capable of reflection. Hexagonal blende gave no effect. Later[1] these sulphides failed to work and could not be revived by heating or cooling, or by exposing to daylight, magnesium light, or X-rays.[2] Finally he excited the sulphides and hexagonal blende by sparks until they were strongly phosphorescent and still they failed to work. Troost[3] had a similar experience with the hexagonal blende, finding that although new crystals were effective the old ones failed. No explanation has been given of this curious phenomenon.

McKissick[4] obtained photographic effects through black paper from lithium chloride in solution, barium sulphide, calcium sulphate, quinine chloride, quinine sulphate, sugar, chalk, glucose, sodium, tungstate, sterein, uranium acetate, and ammonium phospho-molybdate. These substances were exposed to the sun for two hours and then put in the dark. The time of exposure of the plates varied from 48 to 72 hours. It has been found that black paper is not a sufficient protection against chemical action; whether any further precautions were taken is not stated by the writer.

The radiation from uranium and its salts differs from the radiation from other salts in the persistence of the emission of the rays when the substances are kept in darkness. On the other hand, the radiation from the sulphides of calcium resembles the uranium radiation in the fact that they can both be reflected and will both penetrate aluminum and glass. Further than this the relation between the two classes of radiation has not yet been determined.

Conclusions.

As these rays can be reflected, refracted and polarized there can be no reasonable doubt that they are transverse ether waves. Interference alone is left to be established to confirm this, but owing to the extreme feebleness and short wave-length it is doubtful whether it can be shown. The transmission of these rays by metals indicates that the wave-length must be shorter than any ultra-violet waves which have been obtained from any light source.

The radiation seems to be of the nature of a persistent phosphorescence. But the source of energy of this radiation is at present unknown.

These rays, like X-rays, are not homogeneous.

They have all the properties that X-rays possess, such as photographic action, exciting fluorescence, making gases conductors, augmenting the formation of fogs, and exciting thermo-luminescence. In every case the similarity is very striking. The principal difference we find between the

[1] C. R., 122, p. 693.
[2] C. R., 122, p. 766.
[3] C. R., 122, p. 694.
[4] Lond. Elect., 38, p. 313.

two is the absence of regular reflection and refraction of X-rays. When we remember that for very short transverse waves there should be no regular reflection or refraction, a very strong argument as to the character of X-rays is afforded. This leads us to the conclusion that X-rays are transverse ether waves, but very short, shorter than Becquerel rays.

Attempts to obtain diffraction and interference of X-rays have so far proved unsuccessful. But this does not warrant the statement that interference does not exist. If X-rays are extremely short waves, diffraction and interference will be obtained only with very great care, probably by methods better than now known.

There remains the subject of the polarization of X-rays. Polarization can be tested only by reflection or by unequal absorption of rays polarized in different planes. If any substance gives unequal absorption, it should also give double refraction. If double refraction exists, one, at least, of the indices will not be unity, and we should, therefore, have reflection. It follows that if no substance reflects X-rays, polarization cannot be tested. Therefore this failure to detect polarization cannot be used as an argument for its absence. The failure is only what should be expected from extremely short waves.

The similarity in the behavior of X-rays and Becquerel rays certainly presents a strong argument in favor of the theory that X-rays are short transverse ether waves; and, in the opinion of the author, it presents the strongest of the arguments in favor of this theory.

CORNELL UNIVERSITY.

NEW BOOKS.

Lehrbuch der Experimentalphysik. By A. WUELLNER. Fifth edition. Vol. 3, Electricity and Magnetism. pp. xvi+1414. Leipzig, B. G. Teubner, 1897.

It is both interesting and instructive to compare the new edition of Wuellner's Handbook of Physics with the earlier editions of that standard treatise. Electricity and Magnetism, which occupied the fourth volume of all the early editions, has been promoted as it were to the third volume of the edition of 1897, and the volume has grown in bulk, with the growth of the science, from a thousand pages in the edition of 1875, to over fourteen hundred pages. The general form and make-up of the book is but little changed. He who is familiar with the older editions has no difficulty in finding his way through the new one. He notes with approval or disapproval, according to his point of view, the adoption of the new German spelling and the introduction here and there of new matter; but upon the whole the appearance of the book is an entirely familiar one. We note with regret that the excellent custom of writing all the digits beyond the decimal point in a numerical quantity, such as $3.^{1416}$, in smaller type has been abandoned.

The arrangement of Wuellner's volume on Electricity and Magnetism is, as well known, an unusual one. Magnetism occupies the first chapter and this is followed, not by current electricity which would seem the logical arrangement in a treatise which attacks the subject from the magnetic point of view, but by frictional electricity. This arrangement of material has been followed in the new as it has in all the earlier editions. In comparing the new volume with the third edition (1875) one notices the insertion of an introductory chapter on the potential function and potential. This topic, in the older editions, was treated after the completion of magnetism. The treatment has been generalized and the notation changed, and it has been extended to about double its former length, namely to forty-two pages instead of twenty-one pages of the text.

Chapter 1, which deals with magnetism, opens in the same way as in the edition of 1875. Poisson's theory, however, is treated more briefly. The section on Gauss' method for the determination of the period of vibration of a magnet, is identical with that of 1875. There is no suggestion of the use of the chronograph, although in moderate determina-

tions this instrument is frequently employed. The same numerical example is used to illustrate the method as in the earlier editions, but the results are given in c.g.s. units. As though afraid that this modernization of the classical results of Gauss might put him out of touch with some of his old-fashioned readers, the author proceeds to derive the value of g from the length of the second pendulum in Goettingen and straightway converts the value of the moment of the magnet into what he terms the *ordinary units of mechanics;* and yet the c.g.s. system yields us values which are precise multiples of those obtained by the great pioneer in the use of absolute measures—Gauss.

The subsequent chapters of the work have been more completely revised than the chapter of Magnetism and more entirely new matter has been introduced. We find under Frictional Electricity, for example, a discussion of the discharge of zinc plates by light, in which the researches of Hallwachs and Righi are briefly described; there are likewise new articles on the electric behavior of parallel conducting surfaces; on electric images; on the determination of the dielectric constant, which is treated in a full and very excellent manner; on Maxwell's theory of dielectrics; on absolute measurements of the potential of discharge, etc. The changes which time has brought about in the domain of electricity has compelled certain omissions, such as that of the article on the velocity of propagation of electricity and of occasional statements which are no longer warranted. It is interesting to note the omission, for example, of the statement contained on page 158 of the edition of 1875 to the effect that determinations of electric quantity in absolute measure are necessary only in the most exceptional cases, and that up to the time of the writing of that edition they had indeed only been used in a single important instance, namely by Kohlrausch and Weber in their electrodynamical investigations.

It is impossible within the limits of this notice to give even a bare list of the new subjects which have been introduced into this edition. There are excellent accounts of Nernst's theory of the electromotive forces in electrolytes, and of the various investigations which have been made upon the Hall-effect. The sections upon the magnetism of iron have been extended so as to include hysteresis and there are many other important additions to the volume.

In spite of this new material which has increased the bulk of the volume by nearly half, and in spite of the hundreds of references to the newer work which the book contains, the new edition of Wuellner cannot be said to be a modern treatise in the sense in which Winkelmann's Handbuch, for example, is modern. The things upon which stress is laid and to which the reader's attention is almost exclusively directed are the classical works of the masters of the first half of the century. The elec-

tricity and magnetism which Wuellner's volume represents is the science of Faraday, of Gauss and Weber, of Coulomb, and of Ampère, and their contemporaries. Nor is this characteristic to be regretted. Of the writing of new books on electricity there is no end, and in the attempt to describe all that is new and important in this growing science there is a tendency to forget the extraordinary labors of the older investigators.

When we open Wuellner to read about the voltaic cell it is the batteries of Daniel, of Grove, and of Bunsen which are given the first place and the most complete description. The Clark cell, in spite of the enormous amount of labor which has been done upon it and its consequent preëminence as a standard of electromotive force, is treated in the most cursory manner. If we open to the section of galvanometers we find the instruments of Helmholtz and of Gaugain given the chief place, while Wiedemann's galvanometer of thirty years ago may he said to represent the most advanced type of sensitive instruments. In spite of these characteristics, which stamp Wuellner's treatise as belonging to the old school, the work is one so admirably planned and carried out that it will long hold its place among the standard treatises. The preservation, almost untouched of its original form, will make the work in its new edition all the more welcome to those who were brought up upon Wuellner and who know the earlier editions from cover to cover. The references to the newer literature, together with the very clear handling of such new material as has been added, give to the new edition a value fully equal in its day to that which any of the earlier editions possessed in theirs.

E. L. NICHOLS.

Principles of Alternating Current Working. ALFRED HAY, B. Sc. Biggs & Co., Fleet Street, London, E. C.

The work is an elementary treatment of the subject of alternating currents and alternating current machinery, designed for readers whose mathematical knowledge is limited to algebra and trigonometry.

After some preliminary mathematical definitions, and an exposition of the properties of harmonic motion and curves, and vector quantities, the author takes up in the following eight chapters the general subject of alternating current flow in inductive circuits, and discusses in detail self induction and allied quantities, harmonic analysis as applied to alternating currents, determination of electro-motive force and current waves, the measurement of alternating currents, electro-motive forces, and power.

In Chapters XI. and XII. is taken up the subject of capacity and its effect on alternating current flow when used with resistance and with self induction. Then follows a treatment of mutual induction and a discus-

sion of the coreless transformer according to Maxwell's method (simplified). The remainder of the book is devoted to the simple theory of alternating current machinery, including parallel running of alternators, synchronous motors, polyphase generators and induction motors, and the measurement of power in polyphase circuits.

At the end of each chapter are remarks directing the attention of the reader to approximations that are often made in the text of the chapter for simplicity, and referring him to various sources for a fuller knowledge of the subject. In addition are numerous well-chosen problems that would serve admirably to fix ideas.

It is to be regretted that the author has not adopted the simple applicable word "virtual" for the square root of mean square values of current and electro-motive force instead of the long expression "root mean square." An important omission seems to be an absence of any reference to the rotary converter—a machine that has so largely come into use at the receiving end of transmission systems. Again, the author fails to bring out the importance of the static transformer and the uses to which it is put.

On the other hand, many subjects that are ordinarily discussed in the language of calculus are made remarkably clear by the simple use of algebra, and more or less originality seems to exist in his treatment of many subjects. On the whole the book is very well adapted to the class of readers for which it was written.

R. W. QUICK.

GEORGIA SCHOOL OF TECHNOLOGY, ATLANTA, GA.

The Elements of Electric Lighting. Ninth edition. By PHILIP ATKINSON. pp. vi+279. New York, D. Van Nostrand Company. 1897.

Although it has passed through many editions, this latest edition is not materially different from the earlier editions of ten years ago. The arrangement of the book is unchanged, the chapters being identical with the exception that chapters III. and IV. have their relative positions interchanged. The illustrations in the main are those familiar ones which for a decade have been looked upon as standard in works of this class. Certain changes have been made in the book to include something of recent progress. On comparing it, however, with the fifth edition (1889) we notice an exact identity in a large number of the pages throughout the book and minor changes and additions which increases its volume by but twenty pages. The simplicity of style and lack of depth in treatment give popularity to the book, and this latest edition will be perused by many readers who, having no acquaintance with the subject, wish its general and not too technical exposition.

B.

Examples in Electrical Engineering. By SAMUEL JOYCE. 8vo. pp. 240. New York, Longmans, Green & Company, 1896.

This book contains well selected exercises in electrical engineering, including problems pertaining to direct currents, the magnetic circuit and alternating currents. The portions of the book which the reviewer has examined have shown great care and accuracy in preparation, a judiciousness in selection and a directness and clearness in presentation. In addition to the many carefully worked out examples throughout the book over 150 examples are appended with answers, but without solution. Various tables and constants conclude the book, together with eleven pages of tabular data for the cyclic changes of magnetic induction and magnetizing force, together with permeability and hysteresis loss for different maximum values for the usual grades of iron.

F. BEDELL..

The Potentiometer and its Adjuncts. By W. CLARK FISHER. pp. x+194. London, the Electrician Company.

This book deals chiefly with the Crompton potentiometer. The book has been made especially valuable for one employing this particular instrument; and yet this specific treatment has not materially subtracted from the general value of the book. The work has been expanded from articles contributed by Mr. Fisher to the technical press, but contains some added material. The book is a useful one, treating thoroughly, as it does, its particular subject and containing such allied material as will add to the usefulness of the book. As examples, we may refer to the chapters on "Standards of E. M.F.," "Resistance Alloys," and "Platinum Thermometer."

F. B.